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REDUCED ORDER OPTIMAL CONTROL SYNTHESIS OF A CLASS OF NONLINEAR DISTRIBUTED PARAMETER SYSTEMS USING SINGLE NETWORK ADAPTIVE CRITIC DESIGN

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Abstract. *A computational tool is presented in this paper for the optimal control synthesis of a class of nonlinear distributed parameter systems. This systematic methodology incorporates proper orthogonal decomposition based basis function design followed by Galerkin projection, which results in a low-dimensional lumped parameter model. The optimal control problem in the reduced lumped parameter framework is then solved following the philosophy of recently developed 'single network adaptive critic (SNAC)' neural networks. This time domain solution is then mapped back to the distributed domain, which essentially leads to a closed form solution for the control variable in a state feedback form. Finite-element based numerical simulation results are presented for a one-dimensional benchmark nonlinear heat conduction problem.*
Keywords: Adaptive critic, Single network adaptive critic, SNAC, Optimal control, Distributed parameter system control, Proper orthogonal decomposition, Temperature control

1. **Introduction.** In contrast to the lumped parameter systems, Distributed Parameter Systems (DPS) are governed by a set of Partial Differential Equations (PDEs). An interesting brief historical perspective of the control of such systems can be found in [8]. There exist infinite dimensional operator theory based methods for the control of distributed parameter systems. While there are many advantages, these operator theory based approaches are mainly limited to linear systems [6] and some limited class of problems like spatially invariant systems [2]. Such a control design approach is known as "design-then-approximate". Even though the resulting controller from this approach retains most of the information of the associated full-order controller, it is not completely free from errors because of the approximations involved in its implementation. Another control design approach is "approximate-then-design", where the PDEs describing the system dynamics are first approximated to yield a finite dimensional approximate model. This approximate system is then used for controller synthesis. In this approach, it is relatively easy to design controllers using various concepts of finite-dimensional control