

## OPTIMAL LINEAR FILTERING FOR SYSTEMS WITH MULTIPLE STATE AND OBSERVATION DELAYS

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Received November 2006; revised March 2007

**ABSTRACT.** *In this paper, the optimal filtering problem for linear systems with multiple state and observation delays is treated proceeding from the general expression for the stochastic Ito differential of the optimal estimate, error variance, and various error covariances. The paper treats the most general case of multiple delays in both state and observation equations, which are allowed to be different from each other. The resulting system of equations for determining the filter gain matrix consists, in the general case, of an infinite set of equations. It is however demonstrated that a finite set of the filtering equations can be obtained in the particular case of equal or commensurable delays in the observation and state equations. In the example, performance of the designed optimal filter for linear systems with state and observation delays is verified against the best Kalman-Bucy filter available for linear systems without delays.*

**Keywords:** Filtering, Stochastic system, Time-delay system

**1. Introduction.** The optimal filtering problem for linear system states and observations without delays was solved in 1960s [1], and this closed-form solution is known as the Kalman-Bucy filter. However, the related optimal filtering problem for linear states with delay has not been solved in a closed form, regarded as a closed form solution a closed system of a finite number of ordinary differential equations for any finite filtering horizon. The optimal filtering problem for time delay systems itself did not receive so much attention as its control counterpart, and most of the research was concentrated on the filtering problems with observation delays (the papers [2, 3, 4] could be mentioned to make a reference). A few particular cases, the optimal filtering problems for linear systems with state