

## TWO FUZZY ZEROS, TWO FUZZY UNITS

MILAN MAREŠ

ÚTIA AV ČR

P. O. Box 18, 182 00 Praha 8, Czech Republic  
mares@utia.cas.cz

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**ABSTRACT.** *It was shown in [4] and some other works that the group properties of algebraic operations over fuzzy quantities are fulfilled in a weakened form, only, and that this weakening is based on the concepts of fuzzy zero, fuzzy unit and weak fuzzy equivalences using these two concepts. The fuzzy zero and fuzzy unit defined in [4] as relatively wide classes of special fuzzy quantities, namely of the symmetric and transversible ones, respectively. Here, we show that fully equivalent results can be derived even if the “zero” and “unit” classes are essentially narrowed.*

**Keywords:** Fuzzy set, Fuzzy quantity, Group operation, Fuzzy zero, Fuzzy unit, Algebraic equivalence between fuzzy quantities

**1. Introduction and Preliminaries.** The fuzzy set theory [8] offers effective tools for processing vagueness of human natural languages in a wide spectrum of applications. Fuzzy quantities (cf.[4]) and their special case, fuzzy numbers (see, e. g., [1, 2]), are defined as fuzzy subsets of the real line, and represent a model of vague quantitative data. In accordance with the modern fields of application of fuzzy set theoretical procedures, the processing of vague quantitative information becomes significant for realistic models of economic, social and behavioral phenomena. The fuzzy quantities are adequate to the mathematical representation of subjectivity, uncertainty and impression of many real data sources. It means that effective processing of such data has to respect their uncertainty and to specify the degree in which the input vagueness is transformed into the output one (see, e. g., [9, 10]).

The vague quantitative (even numerical) data appear in practical situation quite frequently, and it is often desirable to manage their processing, even when they appear on the input of classical methods, known, e. g., from [3], [7] or [11].

The algebraic operations over fuzzy numbers were summarized, e. g., in [1] and [2]. It is well known that, however fuzzy numbers generalize their crisp counterparts, their algebraic properties are mutually different. The roots of those differences, as well as a formal weakening of some concepts leading to the fulfilment of group properties, are presented in [3] and [4]. In this paper, we study and discuss an alternative approach to such weakening and show that its impact on the algebra of fuzzy quantities is identical with the one suggested in [5] and [4].

The alternative approaches regard two pairs of concepts – the concept of fuzzy zero (based on the symmetry of membership function) and the additive equivalence following from it form one pair, the concept of fuzzy unit (based on the transversibility of membership function [4, 5]) and the multiplicative equivalence following from it form the other pair. The methods of the analysis of both pairs are equivalent.