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NONLINEAR MATHEMATICAL PROGRAMMING FM-BEM FOR THE ELASTIC FRICTIONAL CONTACT SYSTEM

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Abstract. *This paper presents a new mathematical model and a kind of Incomplete Generalized Minimal Residual (IGMRES(m)) algorithm for the highly nonlinear system of frictional contact. The nonlinear programming Fast Multipole Boundary Element Method (FM-BEM) is developed for 3-D elastic contact with friction. Numerical experiments have shown that the optimized programming model for the node-to-surface contact with friction and the proposed IGMRES(m) algorithm could dramatically reduce the times of iteration and improve the computational efficiency with ensured numerical accuracy.*
Keywords: FM-BEM, Nonlinear mathematical programming, IGMRES(m) algorithm

1. **Introduction.** Boundary Element Method (BEM) has been developed as an efficient method following the Finite Element Method (FEM) in recent several decades. BEM has attractive advantages of high accuracy, dimension reduction, and is especially suitable for dealing with the problems related to the infinite or semi-infinite domain, and the problems related to singularity or high gradients. But the conventional BEM is not capable of dealing with complex engineering problems in practice, because the matrix of the resulted algebraic equation is dense and asymmetric, the operations increase in $O(N^3)$ and the memory requirement increases in $O(N^2)$, N is the number of unknowns.

In 1980s and 1990s, Rokhlin and Greengard presented a Fast Multipole Method (FMM) with $O(N)$ operations and memory requirement [2,3]. Combined with the FMM, the BEM was finally developed into a new kind of FM-BEM, which was quite efficient to deal with large-scale engineering and scientific problems. In recent years, the FM-BEM has attracted more and more researchers and has been successfully applied to the field of nonlinear problems, computational mechanics, electro-magnetic field analyses and some other fields [1, 4-11]. Its theory and application prospects will be extremely extensive.

For the elastic frictional contact problems, nonlinear factors occur on the contact surface [12-14]. The contact constraints are highly nonlinear, which results in time-consuming complicated iterations and enlarged solution scale. In order to solve this difficult problem well, a nonlinear mathematical programming method is newly presented to linearize the nonlinear contact constraints, and an optimized mathematical model is established for the frictional contact system, which is suitable for large-scale numerical computation of elastic frictional contact FM-BEM. To accelerate the solution procedure, a kind of new IGMRES(m) algorithm is proposed by using the truncation technology [12,15,16]. It constructs new recursion formulae by only using some of the calculated vectors to compute the following vectors, which could greatly reduce the computation and memory requirement. The FMM is used to compute the product of a matrix and some vectors.