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OPTIMAL FILTERING FOR LINEAR SYSTEMS OVER POLYNOMIAL OBSERVATIONS

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Abstract. *In this paper, the optimal filtering problem for linear system states over polynomial observations is treated proceeding from the general expression for the stochastic Ito differential of the optimal estimate and the error variance. The general case of nonlinear polynomial observations is treated. As a result, the Ito differentials for the optimal estimate and error variance corresponding to the stated filtering problem are first derived. The procedure for obtaining a closed system of the filtering equations for a linear state over observations with any polynomial drift is then established. In the example, the obtained optimal filter is applied to solution of the optimal third degree sensor filtering problem, assuming a Gaussian initial condition for the third degree state. The resulting filter yields a reliable and rapidly converging estimate.*
Keywords: Filtering, Stochastic system, Nonlinear polynomial observations

1. **Introduction.** Although the general optimal solution of the filtering problem for nonlinear state and observation equations confused with white Gaussian noises is given by the Kushner equation for the conditional density of an unobserved state with respect to observations [1], there are a very few known examples of nonlinear systems where the Kushner equation can be reduced to a finite-dimensional closed system of filtering equations for a certain number of lower conditional moments (see [2, 3, 4, 5] for more details). There also exists a considerable bibliography on robust filtering for the "general situation" systems (see, for example, [6, 7, 8, 9, 10, 11, 12, 13]). Apart form the "general situation," the optimal finite-dimensional filters have recently been designed for certain classes of polynomial system states over linear observations with invertible ([14, 15, 16, 17]) or non-invertible ([18, 19]) observation matrix. However, the cited papers never consider filtering problems with nonlinear, in particular, polynomial observations.

This paper presents the optimal finite-dimensional filter for linear system states over polynomial observations, continuing the research in the area of the optimal filtering for polynomial systems. In contrast to the previously obtained results ([14, 15, 16, 17, 18, 19]), the general case of nonlinear polynomial observations is treated. Designing the optimal filter over polynomial observations presents a significant advantage in the filtering theory and practice, since it enables one to address some filtering problems with observation nonlinearities, such as the optimal cubic sensor problem [20]. The optimal filtering problem is treated proceeding from the general expression for the stochastic Ito differential of the optimal estimate and the error variance [21]. As the first result, the Ito differentials for the optimal estimate and error variance corresponding to the stated filtering problem are derived. It is then proved that a closed finite-dimensional system of the optimal filtering equations with respect to a finite number of filtering variables can be obtained for a polynomial observation equation, additionally assuming a Gaussian initial condition for the