International Journal of Innovative Computing, Information and Control
Volume 4, Number 2, February 2008
 ISSN 1349-4198
pp. 321–333

ALGORITHMS FOR DESIGNING REDUCED-ORDER FUNCTIONAL OBSERVERS OF LINEAR SYSTEMS

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Received January 2007; revised June 2007

Abstract. *The problem of finding low-dimensional matrices K, L, E, G and M in the coupled matrix equations $LA - EL = GC$ and $F = KL + MC$ is considered in this paper. This problem is of great importance in the design of reduced-order linear functional observers for linear dynamical systems. Based on a parametric approach, two algorithms for designing reduced-order scalar and multiple functional observers are presented. Several numerical examples are provided to illustrate the effectiveness of the proposed design algorithms.*
Keywords: Sylvester matrix equation, Linear functional observers, Partial-state estimation, Linear systems.

1. **Introduction.** In this paper we consider the following problem. Let m, n and r be given positive integers, matrices $F \in \mathbb{R}^{m\times n}$, $A \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{r\times n}$ and the matrix pair (C, A) be observable with $\text{rank}(C) = r$ and $\text{rank}(F) = m$. Find matrices $L \in \mathbb{R}^{p\times n}$, $E \in \mathbb{R}^{p\times p}$, $G \in \mathbb{R}^{p\times r}$, $K \in \mathbb{R}^{m\times p}$ and $M \in \mathbb{R}^{m\times r}$ such that the following coupled matrix equations hold

$$LA - EL = GC \quad (1)$$
$$F = KL + MC. \quad (2)$$

Here $E \in \mathbb{R}^{p\times p}$ is a stable matrix with a low order p.

The generalized Sylvester matrix equation (1) is closely related with many problems in linear control systems theory. Its solution has been well studied by many authors and effective solution methods have been reported (see, for example [2,3,5,14]). The coupled matrix equations (1)-(2) arise particularly in the problem of designing reduced-order observers for multivariate systems. Many papers have been devoted to this subject, for instance [1,4,6,7,10-13] and the references therein. In this paper, our focus is on the solution to the coupled matrix equations (1)-(2) with a low order p.

A linear functional observer estimates only a linear combination of the states or a partial set of the state vector of a system without estimating all the individual states. Such an observer can have a significantly lower order than that of a full-order state observer.