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OPTIMIZATION ALGORITHM FOR COMPUTING EXACT MINKOWSKI SUM OF 3D CONVEX POLYHEDRA

Xijuan Guo[1], Yanli Gao[2], Yong Liu[1] and Lei Xie[1]

[1]College of Information Science and Engineering
Yanshan University
QinHuangdao 066004, P. R. China
xjgou@ysu.edu.cn; { tietongliuyong; leilei84730 }@yahoo.com.cn

[2]College of Hengshui
Heng Shui 053000, P. R. China
hsgaoli791213@yahoo.com.cn

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Abstract. *In 3-dimensions, computing Minkowski sum of two convex polyhedra is a significant geometric operation, it is equivalent to computing the vector sum of all vertices in two convex polyhedra respectively. An optimization algorithm for computing exact Minkowski sum of 3-convex polyhedra is proposed in the paper. According to Regular Tetrahedron Central Projection of convex polyhedron proposed in the paper, we reduce the problem in 3D to 2D, and computing Minkowski sum of two convex polyhedra amounts to computing the overlay of four pairs of planar subdivisions. New algorithm can get exact Minkowski sum of convex polyhedra, and experiment results indicate that new algorithm can reduce the computing time.*

Keywords: Minkowski sum, Regular tetrahedron central projection, Slope diagram, Doubly connected edge list

1. **Introduction.** P and $Q4$ are two closed convex polyhedra in R3, Minkowski sum of P and Q is convex polyhedron M, $M = P \oplus Q = \{p+q | p \in P, q \in Q\}$ [1]. Here $p+q$ is the sum of position vectors p and q, corresponding to points contained in P and Q. Minkowski sum has the comprehensive applications in robot, computer graphics, CAD/CAM, and so on. For example, in robot path planning, there are start point s and the destination point t in the given configuration space, we look for a path from s to t, where a robot does not intersect the motionless obstacles in the workspace [2], Minkowski sum is a significant tool for computing the collision-free paths. In the workspace, people usually simulate arm of robot and obstacles with polyhedra. Suppose that polyhedron P is the obstacle in workspace, and polyhedron R is a robot which is restricted to translation only, computing the configuration space obstacle in 3-dimensions can be reduced to computing the Minkowski sum of P and $-R$, denoted as $P \oplus -R$, where R is the polyhedron R rotated by 180^o.

In 1983, Lozano-Perez [3] firstly considered that Minkowski sum was an important tool for computing collision-free paths in robot motion planning. So far, many methods for computing Minkowski sum of two convex polyhedra have been proposed, the goals are typically to compute the boundary of Minkowski sum and provide some representation of it. In 1993, Ghosh [4] proposed a unified algorithm for computing 2D and 3D Minkowski sums of convex and non-convex polyhedra based on slope diagram representation, computing Minkowski sum amounts to computing slope diagrams of two polyhedra, merging them, and extracting the boundary of Minkowski sum from the merged diagram. Bekker