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NUMERICAL SOLUTION OF OPTIMAL CONTROL FOR SCALED SYSTEMS BY HYBRID FUNCTIONS

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Abstract. *The function with a stretch are expanded by hybrid functions. The direct algorithm for a product of a matrix function and a vector function is presented. The hybrid stretch matrix is given. Furthermore using the operational matrix of integration the scaled systems are solved. The results are applied to the optimal control problem.*
Keywords: Scaled systems, Hybrid functions, Optimal control

1. **Introduction.** Walsh functions [1], block-pulse functions [2], Laguerre polynomials [3], Chebyshev polynomials [4], Legendre polynomials [5] and Fourier series [6] are of orthogonality used to solve various dynamic systems and optimal control problems. If a function approximated has inherent discontinuities, these methods may not model the discontinuities suitably. The dynamics of current collection systems for electric locomotives could be described by scaled systems [7]. Although these systems were more difficult to solve, the authors in the paper [8] solved the scaled systems by Haar wavelets. Motivated by these results, we use hybrid function parallel in [9, 10] to solve the scaled systems. The advantages of this method are that the convergence of the hybrid solution of the systems is faster than that of Haar wavelets solution and that the integration internal can be divided arbitrarily. So the results will model the discontinuities better. The direct algorithms for a product of a matrix function and a vector function and the expression of a function with a stretch and the hybrid stretch matrix are derived by hybrid functions. As a result the scaled systems are reduced to the algebraic equations. Applying the result to the optimal control problem the approximate solutions of the optimal control and state as well as the optimal value of the objective functional are obtained. A numerical example is worked out to illustrate this applicable method.

2. **Preliminaries.** A set of block-pulse function $b_k(t),\ k = 1, 2, \ldots, K$, on the interval $[0, T)$ are defined as

$$b_k(t) = \begin{cases} 1, & t_{k-1} \le t < t_k, \\ 0, & \text{otherwise} \end{cases}$$

where $t_0 = 0$, $t_K = T$ and $[t_{k-1}, t_k) \subset [0, T)$, $k = 1, 2, \ldots, K$.

The Legendre polynomials $L_m(t)$ on the interval $[-1, 1]$ are given by the following recursive formula

$$\begin{cases} L_0(t) = 1,\ L_1(t) = t, \\ (m+1)L_{m+1}(t) = (2m+1)tL_m(t) - mL_{m-1}(t). \end{cases}$$