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THREE POSITIVE SOLUTIONS FOR HIGH-ORDER m-POINT BOUNDARY VALUE PROBLEMS WITH ALL DERIVATIVES

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Abstract. *Utilizing the five functionals fixed point theorem, we prove the existence of at least three positive solutions for* $2n$*th order* m*-point boundary value problems. In order to do so, firstly, we give the associated Green's function and its properties. Then we impose growth conditions on the nonlinear term* f *which ensure the existence of at least three positive solutions. Finally, we give an example to demonstrate our result. Our result extends and improves some of the existing literature.*

Keywords: High-order multi-point boundary value problem, Cone, Positive solution, Green's function, The five functionals fixed point theorem

1. **Introduction and Preliminaries.** The multi-point boundary value problems for ordinary differential equations arise in a variety of areas of applied mathematics and physics. For example, the vibrations of a guy wire of a uniform cross-section and composed of N parts of different densities can be set up as a multi-point boundary value problem (see [18]); many problems in the theory of elastic stability can be handled by the method of multi-point boundary value problems (see [19]). In [6], Il'in and Moiseev studied the existence of solutions for a linear multi-point boundary value problem. Motivated by the result of Il'in and Moiseev [6], Gupta [7] studied certain three-point boundary value problems for nonlinear ordinary differential equations. Since then, more general nonlinear multi-point boundary value problems have been studied by several authors. We refer the reader to [1-5,8-17] and references cited therein.

In paper [1,2], Davis et al. studied the $2m$th Lidstone boundary value problems

$$\begin{cases} y^{(2m)}(t) = f(y(t), y''(t), \ldots, y^{(2(m-2))}(t), y^{(2(m-1))}(t)), \ 0 \le t \le 1, \\ y^{(2i)}(0) = y^{(2i)}(1) = 0, \ 0 \le i \le m-1, \end{cases}$$

where $(-1)^m f : R^m \to [0, \infty)$ is continuous. They obtained the existence of three symmetric positive solutions by applying Leggett-Williams fixed point theorem and the five functionals fixed point theorem, respectively.

In paper [3], using the five functionals fixed point theorem, Zhang et al. proved the existence of positive solutions for the $2m$th order two point boundary value problems

$$\begin{cases} y^{(2m)} = f(y(t), y'(t), \ldots, y^{(2m-2)}(t), y^{(2m-1)}(t)), \ 0 \le t \le 1, \\ y^{(2i)}(0) = y^{(2i)}(1) = 0, \ 0 \le i \le m-1, \end{cases}$$