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EQUIVALENT REPRESENTATIONS OF BI-MATRIX GAMES

Dianyu Jiang

Institute of Game Theory with Applications
Huaihai Institute of Technology
No.59 Cangwu Road, Xinpu, Lianyungang-city 222005, P. R. China
jiangdianyu425@126.com

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Abstract. *A bi-matrix game is said to be rational if the two players' common knowledge contains and only contains these: (1) The two players. (2) For either player, a set of actions. (3) Payoffs received by either player for the pair of the actions. (4) Either of players knowing the principle of maximum entropy. In this paper, we put forward a concept of Neumann-Morgenstern stable set (briefly, N-M stable set) which is a subset of set of all pure Nash equilibria in a bi-matrix game. Give an algorithm of finiding N-M statble set. Prove existence and uniqueness of N-M stable set in a bi-matrix game with at least one pure Nash equilibrium. An ideal game is defined as the game whose two players have the same preferences for all pure situations in the N-M stable. For example, a rational game is an ideal game. We prove that every bi-matrix game is equivalent to one and only one ideal game.*

Keywords: Bi-matrix game, Completely static game, Rational game, N-M stable set, Ideal game, Principle of maximal entropy

1. **Introduction.** The idea of Nash equilibrium is one of the most powerful concepts in game theory. Nash [1,2] proved the existence of mixed strategy equilibria for finite games. From a decision theoretic viewpoint the concept of mixed strategy Nash equilibrium is less compelling than the concept of pure strategy Nash equlibrium (PNE). It is therefore interesting to study PNE in a finite game, under different conditions.

In finite game theory, the simplest model is the so-called bi-matrix game. The game is represented as a two-dimensional matrix, in which the rows represent the pure strategies for one player and the columns those for the other. In each cell is placed a pair of numbers representing the payoffs to the two players if the corresponding pair of strategies is chosen. For set of pure Nash equilibria of a bi-matrix game, there are, mostly, the hands. The first is existence of pure Nash equilibria, such as [3-5]. The second is number of pure Nash equilibria and its estimation, such as [6-10]. The third is structure of set of pure Nash equilibria, such as [11-13]. The fourth is regularity and stability of equilibrium points of bi-matrix games and so forth, such as [14,15].

Except these, we will develop a more important problem, in this paper, is what pure Nash equilibria the two players should prefer. As everyone knows, realization of a pure Nash equilibrium is important and difficult [16]. Our problem can make it is easier that the two players to realize a pure Nash equilibrium.

If a player repeatedly play a zero-sum game, a mixed strategy adds an uncertainty which could confuse his/her enemy [17]. As a result, each player hopes to use his/her mixed strategy with the greatest uncertainty.

In 1948, Shannon introduced the concept of information entropy to describe an uncertainty of a random variable taking value [18].