THE MODIFIED $\mathbf{D}_p$-METRIC SPACE OF FUZZY SET-VALUED RANDOM VARIABLES AND ITS APPLICATION TO VARIANCES

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ABSTRACT. *In this paper, we shall first introduce the definitions of modified* $\mathbf{D}_p$ *metrics for fuzzy set-valued random variables so that the related spaces of fuzzy set-valued random variables are complete. By using a modified* $\mathbf{D}_p$ *metric, we shall define the variance and covariance of fuzzy set-valued random variables and show their application in statistical estimation.*
Keywords: Fuzzy set-valued random variables, Modified $\mathbf{D}_p$-metric, Variance and Covariance

1. **Introduction.** Frequently, we have to deal with the statistical experiments in practice, whose sample values are not real-valued data but vague descriptions of language, i.e. we have to treat the data exhibiting both randomness and vagueness. Hence it is very important to introduce fuzzy set-valued random variables (fuzzy random variables or random fuzzy sets) for establishing mathematical models. The concept of fuzzy set-valued random variable was investigated by researchers by using different ways (e.g. [14], [21]). Here we would like to mention the definition introduced by Puri and Ralescu [21]. They firstly defined a fuzzy random variable as the generalized set-valued random variable or random set (e.g.[1], [5], [17]), and discussed limit theorems of sequences of fuzzy set-valued random variables in [22] and [23]. There are many further research works in this area such as Fukuda [3], Guan and Li [4], Inoue [7], [8], Klement *et. al.* [9], Li *et. al.* [16], [17], Nather [20] and so on.

In classical statistics, as it is well-known, the moments of random variables especially the first and second order play a very important role with many important applications. For instance, Markowitz used the expectation and variance to investigate portfolio selection problem [19]. Readers may refer to [25] for its further discussion by soft method. In the same way, some of the most useful information concerning set-valued random variables as well as fuzzy set-valued random variables can be revealed by its moments. Usually, the expectation (the first order moment) of a fuzzy set-valued random variable is defined by the generalized Aumman Expectation EX (cf. [1], [21]) by using level set method. The properties of the expectation of fuzzy set-valued random variables have been discussed (e.g. [21], [17]). However, only a few attempts are made to define the variance or covariance of fuzzy set-valued random variables and to study their properties, since the space of all subsets of a basic space (e.g. d-dimensional Euclidean space) is not linear with resect to the addition and scalar multiplication. It leds to a big difficult problem: how to define the minus of two subsets? Hukuhara introduced a definition of minus in [6]. But we do not know under what conditions the Hukuhara difference exists in general. So we can not