

TWO POSITIVE SOLUTIONS FOR MULTI-POINT BOUNDARY VALUE PROBLEMS WITH p -LAPLACIAN

WEIHUA JIANG^{1,2}, JIQING QIU² AND YAN SHI³

¹College of Mathematics and Science of Information
Hebei Normal University
Shijiazhuang, 050016, Hebei, P. R. China
weihua.jiang@hebust.edu.cn

²College of Sciences
Hebei University of Science and Technology
Shijiazhuang, 050018, Hebei, P. R. China
qiujiqing@263.net

³School of Industrial Engineering
Tokai University
9-1-1, Toroku, Kumamoto 862-8652, Japan
yshi@ktmail.tokai-u.jp

Received June 2007; revised December 2007

ABSTRACT. *The existence of at least two positive solutions for second order multi-point boundary value problems with p -Laplacian is obtained by utilizing H. Amann fixed point theorem, where the nonlinear term involves the first derivative and may be noncontinuous. The method used in this paper is new.*

Keywords: Cone, Multi-point boundary value problem, Positive solution, H. Amann fixed point theorem

1. Introduction and Preliminaries. The boundary value problems with p -Laplacian arise from the study of the p -Laplacian equation, non-Newtonian fluid theory, and the turbulent flow of gas in a porous medium. And these problems have been studied extensively in the literature by using Guo-Krasnoselskii fixed-point theorem, fixed point index theory, upper and lower solutions method, fixed point theorem due to Avery and Peterson and so on; we refer the reader to [1-7] and references cited therein.

Using a fixed point theorem on a cone, Ji et. al. ([11]) investigated the existence of at least one positive solution for the problem

$$(\varphi_p(u'))'(t) + h(t)f(t, u(t), u'(t)) = 0, \quad t \in (0, 1), \quad (1)$$

$$u'(0) = 0, \quad u(1) = \sum_{i=1}^{m-2} \alpha_i u(\xi_i), \quad (2)$$

where $\varphi_p(s) = |s|^{p-2}s$, $p > 1$, $0 < \xi_1 < \xi_2 < \dots < \xi_{m-2} < 1$, $0 < \alpha_i < 1$, $i = 1, 2, \dots, m-2$, $0 < \sum_{i=1}^{m-2} \alpha_i < 1$, $h(t)$ and $f(t, u, v)$ are continuous.

Using fixed point theorem due to Avery and Peterson, which is a generalization of the Leggett-Williams fixed point theorem, B. Sun et. al. ([8]), H. Feng et. al. ([9]) and Y. Wang et. al. ([10]) studied the existence of at least three positive solutions for the