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H_∞ CONTROL OF LINEAR POSITIVE SYSTEMS: CONTINUOUS- AND DISCRETE-TIME CASES

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Abstract. *This note investigates the H_∞ control problem for linear continuous- and discrete-time positive systems. Instead of using algebraic techniques which have been widely employed for the analysis of positive systems, our development is based on matrix inequalities. With the well-established results of Lyapunov stability theory and nonnegative matrix, sufficient conditions are derived for the existence of desired H_∞ state-feedback controllers that guaranteeing the resultant closed-loop system not only to be asymptotically stable and positive, but also has a desired H_∞ performance. Moreover, even for an unstable and non-positive system, the results obtained can be applied to design a desired H_∞ controller such that the closed-loop system not only is stable and positive but also has an H_∞ performance. Since the conditions obtained are expressed as linear matrix inequalities (LMIs), which can be easily verified by using standard numerical software. Numerical examples are provided to illustrate the proposed design scheme.*

Keywords: H_∞ control, Linear matrix inequality (LMI), Nonnegative matrix, Metzler matrix, Positive systems

1. **Introduction.** In many practical mathematical modeling, there exist such systems, for instance, networks of reservoirs, industrial processes involving chemical reactors, heat exchangers and distillation columns, hierarchical systems, water and atmospheric pollution models, stochastic models, where the state variables must be constrained to be positive (at least nonnegative) [6]. Such systems, known as positive systems, have the peculiar property that any nonnegative input and nonnegative initial state generate a nonnegative state trajectory and output for all times. Nonnegativity of each state vector element for all times of positive systems will bring many new problems, which cannot be solved by using well developed methods for general linear systems. Therefore, seeking new approaches to study this kind of systems has been received many researchers' attention, and many fundamental results have been reported (see, for instance, [5, 11, 14] and the references therein). To mention a few, the stability problem of positive systems is investigated in [6, 11, 14], the stabilization problem is studied in [12], the eigenvalue regions of positive systems are analyzed in [2], and so on (readers are referred to [6, 13] for a detailed account of the recent development in positive systems). It should be pointed out here that the current studies for positive systems are mainly focused on the behavioral analysis. While the control problems to give positivity and performance analysis seem to receive relatively less attention. On the other hand, the current studies are based on model without considering the effect of the external disturbances which often exist in