OSCILLATION AND NONOSCILLATION OF NEUTRAL DIFFERENCE EQUATION WITH POSITIVE AND NEGATIVE COEFFICIENTS

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Abstract. *In this paper, the oscillation and nonoscillation properties of all solutions of the following neutral difference equation with positive and negative coefficients are investigated under the conditions different from the known literature and the given results improve that in the known literature.*
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1. **Introduction.** In this paper, we are going to deal with oscillation and nonoscillation of all solutions of the following neutral difference with positive and negative coefficients

$$\Delta[x(n) - c(n)x(n-r)] + p(n)x(n-k) - q(n)x(n-l) = 0, \quad n = 0, 1, 2, \dots \quad (1)$$

where $c(n)$, $p(n)$, $q(n)$ $(n = 0, 1, 2, \dots)$ are real numbers with $c(n) \geq 0$, $p(n) \geq 0$, $q(n) \geq 0$ and r, l, k are integers with $r > 0$, $l \geq 0$, $k > l + 1$.

The oscillatory and nonoscillatory behavior of Equation (1) was investigated by several authors, [1-4] under the hypothesis

$$\sum_{n=0}^{\infty} [p(n) - q(n-k+l)] = \infty \quad (2)$$

which played a key role in the study of the oscillation of Equation (1). However, Tang and Yu [5] obtained some sufficient conditions for the oscillation and nonoscillation of Equation (1) which do not require condition (2). They used the following known condition

$$c(n) + \sum_{s=n-k+l}^{n-1} q(s) \equiv 1 \quad (3)$$

Our aim in this paper is to deal with the oscillation and nonoscillation of Equation (1) without the usual condition (3), and the given results improve the known results in the above mentioned literatures.