

THE LINEAR QUADRATIC OPTIMIZATION PROBLEM FOR A CLASS OF DISCRETE-TIME STOCHASTIC LINEAR SYSTEMS

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ABSTRACT. This paper investigates the problem of the optimization of a quadratic cost functional along the trajectories of a discrete-time affine stochastic system affected by jump Markov perturbations and independent random perturbations. Both the case of finite time horizon as well as the infinite time horizon are considered. The optimal control is constructed using a suitable solution for a system of discrete-time Riccati type equations.

Keywords: Linear quadratic problems, Discrete-time stochastic systems, Markov chains, Independent random perturbations

1. Introduction. The state space approach for the problem of minimization of a quadratic cost functional along the trajectories of a linear (affine) controlled system has a long history. Such an optimization problem is usually known as "the linear quadratic optimization problem" LQOP. Starting with the pioneer work of Kalman [18] the solution of LQOP is closely related to the existence of some suitable solutions of a matrix differential (difference) equations of Riccati type. In [3, 8, 9, 12] it was shown that the solution of the linear quadratic optimization problem strongly depends upon the class of admissible controls. It was proved that for a given cost functional and for different sets of admissible controls the solution of the optimization problem is constructed either using the maximal solution or the minimal solution of the corresponding Riccati differential equation. In the discrete-time stochastic framework the LQOP was separately investigated for systems with independent random perturbations and systems with Markov perturbations, respectively. Thus, for the case of discrete-time stochastic systems with independent random perturbations we refer to [4, 20, 23], while for discrete-time systems with Markovian switching we mention [1, 2, 4, 5, 14, 15, 16, 19, 21, 22]. Lately has been an increasing interest in the investigation of different control problems related to discrete-time linear stochastic systems corrupted by independent random perturbations and Markovian switching. For the readers convenience we refer to a recent paper [6] where several optimization problems having the cost functional given by the terminal value of the expectation or of the variance of an output are studied. In the present paper we consider an optimization problem asking the minimization of a quadratic cost functional along the trajectories of a discrete-time stochastic system subject to Markovian switching and independent random perturbations. Moreover in the state equations additive stochastic perturbations are also present, which determine the corresponding specific form of the cost functional. The class of admissible controls is at the same time general and different from those which are considered in the above mentioned papers. This class is suggested by the problem of tracking of a bounded or of a periodic signal (a problem which appears naturally in