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ALTERNATIVE OPTIMAL FILTER FOR LINEAR SYSTEMS WITH MULTIPLE STATE AND OBSERVATION DELAYS

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Abstract. *In this paper, the optimal filtering problem for linear systems with multiple state and observation delays is treated using the optimal estimate of the state transition matrix. As a result, the alternative optimal filter is derived in the form similar to the traditional Kalman-Bucy one, i.e., consists of only two equations, for the optimal estimate and the estimation error variance. Thus, this paper designs the optimal mean-square finite-dimensional filter for linear time-delay systems with arbitrary, even non-commensurable delays, in both state and observation equations. This presents a significant advantage in comparison to the previously obtained optimal filter* [1]*, which includes infinite or variable number of covariance equations, unboundedly growing as the filtering horizon tends to infinity. Performances of the two optimal filters are compared in example; the obtained results are discussed.*
Keywords: Filtering, Linear stochastic system, Multiple state, Observation delays

1. **Introduction.** The optimal filtering problem for linear system states and observations without delays was solved in 1960s [2], and this closed form solution is known as the Kalman-Bucy filter. However, the related optimal filtering problem for linear states with delay has not been solved in a closed form, regarding as a closed form solution a closed system of a finite number of ordinary differential equations for any finite filtering horizon. The optimal filtering problem for time delay systems itself did not receive so much attention as its control counterpart, since most of the research was concentrated on the filtering problems with observation delays (the papers [3, 4, 5] could be mentioned to make a reference). A few particular cases, the optimal filtering problems for linear systems with state delay and/or multiple observation delays, have recently been solved in [6, 7, 8, 9, 1]. A Kalman-like estimator for linear systems with observation delay has recently been designed in [10]. The optimal filter for linear systems with multiple state and observation delays, derived in [1], has solved the same filtering problem as the present paper. However, that solution is not free of computational disadvantages: it includes a variable number of covariance equations, which unboundedly grows as the filtering horizon tends to infinity, and the structure of the covariance equations also varies with the number. There also exists a considerable bibliography related to the robust control and filtering problems for time-delay systems (such as [11]–[22]). A number of papers, published in 1970s, were dedicated to some particular optimal filtering problems for time-delay systems (see [23]). Comprehensive reviews of theory and algorithms for time delay systems are given in [24]–[30].

In this paper, the optimal filtering problem for linear systems with multiple state and observation delays is treated using the optimal estimate of the state transition matrix from the current time moment to the delayed ones. In doing so, the employed method