

## ROBUST NASH STRATEGIES BASED ON INTEGRAL SLIDING MODE CONTROL FOR A TWO PLAYERS UNCERTAIN LINEAR AFFINE-QUADRATIC GAME

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**ABSTRACT.** *This paper tackles the problem of a two players linear affine-quadratic Multi-Model differential game affected with matched uncertainties. Based on a recent result that develops a type Robust Nash equilibrium for a multi-modeled linear affine-quadratic game [12], where the players compose a min-max strategy to deal with the parametric uncertainties (unmatched type) in this paper we add a second type of uncertainties of the “matched type”. It is shown that for the case of a two players game, the matched uncertainties can be rejected from the initial time introducing in the strategies an integral sliding mode control term, obtaining an equivalent game dynamic in which the matched uncertainties have been eliminated.*

**Keywords:** Robust Nash equilibrium, Integral sliding mode control, Uncertain LQ games

### 1. Introduction.

1.1. **Antecedents and motivation.** In the last decades significant advances have been made in the studies of the linear quadratic (LQ) differential games affected with some sort of uncertainties (see [2], [3], [5], [11], [12], [17], [19]). They deal with the optimization of multiple control agents (players) dynamics when no one of them can control the decision made by others and each dynamic is affected by the consequences of these decisions. A special interest within the last publications is related with the analysis of different uncertain effects to player’s behaviour. In [2], [3], [5] LQ games with uncertainty scenarios have been considered using the  $\mathcal{H}^\infty$  approach leading to the min-max formulation. The paper [2], as well as [3], really deals with two-person uncertain LQ differential game with uncertainties which may “*play against*” players. This property allows to reformulate the problem of a Nash-equilibrium finding as a saddle-point calculation that serves a lot for the construction of a numerical procedure (in the same manner as in [5]). In the paper of [19] the author proposes a Robust Nash equilibrium concept where the game uncertainty is represented by a malevolent input which is subject to a cost penalty or a direct bound.

Then, the  $\mathcal{H}^\infty$  theory of robust control is once again used by the authors to design the robust strategies for all players. A different formulation of a Robust Nash equilibrium was developed in [12], where it is considered a class of multi-model game given by a system of