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POSITIVE SOLUTIONS FOR NONLINEAR nTH-ORDER m-POINT BOUNDARY VALUE PROBLEM WITH THE FIRST DERIVATIVE

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Abstract. *This paper gives the sufficient conditions of the existence of positive solutions for higher order multi-point boundary value problem with the first derivative. The arguments are based upon a specially constructed cone and the fixed point theory in cone. An example is given to demonstrate our result. Our result improves and generalizes some of the existing literature.*
Keywords: Green's function, Fixed point theorem, m-point boundary value problem

1. **Introduction and Preliminaries.** The multi-point boundary value problems for ordinary differential equations arise in a variety of areas of applied mathematics and physics. For example, bridges of small size are often designed with two supported points, which lead to a standard two-point boundary value condition, and bridges of large size are sometimes contrived with multipoint supports, which correspond to a multipoint boundary value condition (see [8]); the vibrations of a guy wire of a uniform cross-section and composed of N parts of different densities can be set up as a multi-point boundary value problem (see [9]); many problems in the theory of elastic stability can be handled by the method of multi-point boundary value problems (see [10]). The study of multipoint BVPs for linear second-order ordinary differential equations was initiated by Il'in and Moiseev [11]. Since then many authors have studied more general nonlinear multipoint boundary value problems. We refer readers to [1-7,12] and the references cited therein.

In [1], Guo and Ge studied the existence of a positive solution for the following boundary value problem

$$\begin{cases} x''(t) + f(t, x(t), x'(t)) = 0, \ 0 \le t \le 1, \\ x(0) = 0, \ x(1) = \alpha x(\eta), \end{cases}$$

where f is a nonnegative continuous function, $\alpha > 0, \ \eta \in (0, 1)$ and $\alpha\eta < 1$.

In [2], Eloe and Ahmad studied the existence of positive solutions for nth-order three-point boundary value problem

$$\begin{cases} u^{(n)}(t) + a(t)f(u) = 0, \ 0 < t < 1, \\ u(0) = 0, u'(0) = 0, \ldots, u^{(n-2)}(0) = 0, \ \alpha u(\eta) = u(1), \end{cases}$$