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STUDYING DISTRIBUTION FUNCTIONS OF FUZZY RANDOM VARIABLES AND ITS APPLICATIONS TO CRITICAL VALUE FUNCTIONS

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Abstract. *In many fuzzy random optimization models, the objectives and constraints may consist of some distribution functions and critical value functions of prescribed fuzzy random variables. Therefore, we need to analyze the properties of those distribution functions and critical value functions so as to design more precise algorithms to solve such optimization problems. In this paper, we deal with the analytical properties of distributions functions of fuzzy random variables and discuss its applications to critical value functions. We first establish some continuity theorems for distribution functions of fuzzy random variables, which characterize the properties of right continuity, left continuity and continuity, respectively. Then, applying those continuity theorems, we study the properties of critical value functions of fuzzy random variables. The results obtained in this paper are useful in fuzzy random programming models.*
Keywords: Fuzzy random variable, Distribution function, Critical value function, Continuity theorem, Fuzzy random optimization

1. **Introduction.** On the basis of possibility theory [1, 18, 22, 26], fuzzy programming models provide an important aspect in handing practical decision-making problems. They deal with a situation when the input data are of linguistic uncertainty or fuzziness. The readers who are interested in fuzzy optimization models may refer to Inuiguchi and Ramik [4], Lee and Li [10], Liu and Zhu [17], Tanaka and Watada [25], and Zhi and Watanabe [27].

However, in a practical decision-making process, we often face a hybrid uncertain environment where randomness and fuzziness coexist. For this case, it is insufficient to use only one of random variable or fuzzy variable to treat them. Fuzzy random variable was introduced by Kwakernaak [8, 9] to depict the phenomena in which fuzziness and randomness appear at the same time. Since then, its variants as well as extensions were presented by other researchers for different purposes, e.g., Krätschmer [5], Kruse and Meyer [7], López-Diaz and Gil [14], Liu and Liu [15], and Puri and Ralescu [19]. Using fuzzy random variables, a number of fuzzy random optimization models are proposed, such as Luhandjula [6], Liu [12], Liu and Liu [16], Yager [23], and Yazenin [24].

In many fuzzy random optimization models, the objectives and constraints may consist of some distribution functions or some critical value functions of prescribed fuzzy random variables. Let's consider two fuzzy random optimization models. One is the following *Fuzzy Random Minimum Risk Problem* (see Liu and Liu [16]):