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DECISION ANALYSIS OF FUZZY PARTITION TREE APPLYING FUZZY THEORY

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Abstract. *We often use fuzzy graph to analyze inexact information such as sociogram structure ([1,2]). Concerning the hierarchical cluster analysis of a fuzzy graph ([3-5] and [11]), the number of clusters may have to be decided in the actual cluster analysis. In other word, we would like to decide the optimal level with a partition tree. Concerning this problem, the steepest decent method in the multivariate analysis and AIC method in the statistical analysis have been designed by us ([6,10]). But, the steepest decent method could produce local limited solution problem and AIC method which is reasonablely based on the statistical inference needs a lot of samples. For these reasons, we would propose the evaluation method based on Fuzzy Decision which could obtain the optimal level even if there are not so many samples. In this paper, we would proceed to explain the practical effectiveness of our method through the cases in sociometry analysis.*
Keywords: Fuzzy graph, Partition tree, AIC (Akaike's information criterion), Fuzzy decision, Optimal level

1. Introduction.

A fuzzy graph G is defined by

$$G = (V, Y),\ V = \{v_i; 1 \leq i \leq n\}, Y = \{y_{ij}; 0 \leq y_{ij} \leq 1\} \tag{1}$$

where y_{ij} is the fuzziness of the arc from the node v_i to the node v_j.

If $f_{ij} = f_{ji}, f_{ii} = 1$ then G signifies the similarity relation among nodes. Here, by analyzing the fuzzy graph G, we have a partition tree P. In the cluster analysis of fuzzy graph, it is important and difficult problem how to decide the optimal cutting level as to a partition tree. By now, the steepest descent method in the multivariate analysis and AIC method in the statistical analysis have been designed. But, the steepest decent method could produce local limited solution problem and AIC method which is reasonablely based on the statistical inference needs a lot of samples. Here, we shall consider the fuzzy decision method by constructing the evaluation functions that pay attention to each level of the partition tree.

2. Steepest Decent Method.

We would define

$$R_z = \{G_1, G_2, \cdots G_m\}, (1 \leq m \leq n, 0 \leq z \leq 1) \tag{2}$$

as the division of $G, (|G| = n)$. For example, in Figure 3.

$$R_{0.62} = \{\{1, 4, 7\}, \{2, 3\}, \{11, 6\}, \{9, 10, 5, 8\}\}$$

The number of clusters of R_z could be defined as $x_{(z)}$ concerning the partition tree. In general, the more level z approaches to 1, the more the number of clusters $x_{(z)}$ increases.