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ROBUST $\mathcal{H}_2$ OUTPUT FEEDBACK CONTROL FOR A CLASS OF TIME-DELAY SYSTEMS

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Abstract. *In this paper, the problem of designing an $\mathcal{H}_2$ output feedback controller for a class of uncertain systems with time-varying delays is investigated. These systems are evolve in the continuous-time space against a wide class of parametric uncertainties which belong to a convex bounded polytopic domain. The objective is to derive tractable synthesis conditions for the robust feedback design. Two design methods are established: the first uses bounds on the delay pattern and the second utilizes instantaneous delay measurements. All the developed results are cast in the format of linear matrix inequalities (LMIs) and numerical examples are presented.*
Keywords: $\mathcal{H}_2$ control, Time-delay systems, Output feedback control, Convex polytopic uncertainties, LMIs

1. **Introduction.** It becomes increasingly apparent that delays occur in physical and man-made systems due to various reasons including finite capabilities of information processing among different parts of the system, inherent phenomena like mass transport flow and recycling and/or by product of computational delays [15]. Stability and stabilization of time-delay systems have been topics of recurring interest over the past decades [5,6,7,8,18,19,20,21,22] since delays are often the main causes of instability and poor performance of dynamic systems. These include, but not limited to, chemical processes, long transmission lines, communication networks, see [3,15] and their bibliographies. Considerable discussions on delays and their stabilization/destabilization effects in control systems have been reported, see [4,9,12,16,17] and the references cited therein. Existing stabilization methods include, but not limited to, $\mathcal{H}_\infty$ control [7,11] and guaranteed cost control using feedback control techniques [16,22]. Little attention has been given to $\mathcal{H}_2$ optimal control using output feedback.

The purpose of this paper is to fill up this gap by examining robust feedback stabilization for a class of uncertain time-delay systems. The parametric uncertainties belong to a convex bounded polytopic domain. We investigate the problem of designing an $\mathcal{H}_2$ output-feedback controller. Both weak- and strong-delay dependent cases are treated and all the results are cast in the form of convex optimization over a system of linear matrix inequalities (LMIs). Simulations results on typical examples are provided illustrate the analytical developments.