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# A NEURAL NETWORK FOR SMOOTH NONCONVEX SADDLE POINT PROBLEMS

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Abstract. *This paper introduces a neural network for solving a wider class of smooth nonconvex saddle point problems. The nonconvexity and the mixed constraints are the two significant characters of the problem considered in this paper. Under a suitable assumption on the constrained set and a proper assumption on the objective function $V(x, y)$, it is proved that for a sufficiently large penalty parameter, there is unique global solution to the neural network, and the unique trajectory of the neural network can reach the feasible region in finite time and stays there thereafter. Moreover, we can prove that the trajectory of the neural network converges to the set consisted by the equilibrium points of the network, whose elements are all the critical saddle points of the problem. Moreover, we prove the coincidence between the unique solution to the neural network modeled by a differential inclusion and the "slow solution" of it. Furthermore, one illustrative example shows the correctness of the results in this paper, and the good performance of the proposed neural network.*

**Keywords:** Neural network, Nonconvex saddle point problem, Differential inclusion, Critical saddle point

## 1. Introduction.

Consider the following saddle point problem:

$$\begin{aligned} \min_x \max_y \quad & V(x, y) = V_1(x) - V_2(y), \\ \text{subject to} \quad & H_1(x) \le 0,\ H_2(y) \le 0,\ B_1x + B_2y - b \le 0, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $V_1(x) : \mathbb{R}^n \to \mathbb{R}$ and $V_2(y) : \mathbb{R}^m \to \mathbb{R}$ are smooth functions, $H_1(x) = (H_{11}, H_{12}, \cdots, H_{1p})^T : \mathbb{R}^n \to \mathbb{R}^p$, $H_2(y) = (H_{21}, H_{22}, \cdots, H_{2q})^T : \mathbb{R}^m \to \mathbb{R}^q$, where $H_{1i} : \mathbb{R}^n \to \mathbb{R}$, $H_{2j} : \mathbb{R}^m \to \mathbb{R}$, $i = 1, 2, \cdots, p$, $j = 1, 2, \cdots, q$ are smooth convex functions, $B_1 \in \mathbb{R}^{r\times n}$, $B_2 \in \mathbb{R}^{r\times m}$ and $b \in \mathbb{R}^r$.

It is well-known that the constrained saddle point problem which is a kind of constrained optimization problem provides a useful reformulation of optimality conditions and also arises in a variety of engineering and economic contexts including game theory, military scheduling, automatic control, and so on. Thus, solving the saddle point problem (1) is very useful for the constrained optimization problems. Particularly, saddle point criteria is a sufficient optimality condition for constrained optimization problems. Facing the convex

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