

## A PARAMETRIC ANALYSIS FOR SINGLE MACHINE SCHEDULING WITH PAST-SEQUENCE-DEPENDENT SETUP TIMES

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**ABSTRACT.** Recently, the single machine scheduling problem with past-sequence-dependent (*p-s-d*) setup times is getting more attentions from academic researchers and industrial practitioners. The past-sequence-dependent setup times are proportional to the length of already scheduled jobs. It is shown that for a number of objective functions this scheduling problem can be solved in  $O(n \log n)$  time. In this paper, we extend the analysis of the problem with the total absolute difference in completion times (*TADC*) as the objective function. This problem is denoted as  $1/s_{psd}/TADC$  in [1]. Let  $s_{[j]}$  and  $p_{[j]}$  be the setup time and processing time of a job occupying position  $j$  in the sequence respectively, and  $s_{[j]}$  is defined as  $s_{[j]} = \gamma \sum_{i=1}^{j-1} p_{[i]}$ , where  $\gamma$  is a normalizing constant. In this paper, we present a parametric analysis of  $\gamma$  on the  $1/s_{psd}/TADC$  problem. We show analytically the number of optimal sequences and the range of  $\gamma$  in which each of the sequence is optimal. We prove that the number of optimal sequences is  $\{1 + \sum_{k=1}^x (2k)\}$  if  $n$  is odd, and  $\{1 + \sum_{k=1}^x (2k-1)\}$  if  $n$  is even. The value of  $x$  is  $\lfloor \frac{n}{2} \rfloor - 1$  when  $n$  is odd, and  $x$  is  $\frac{n}{2}$  when  $n$  is even. The number of optimal sequences depends only on  $n$ , the number of jobs, and not on  $\gamma$ . We also show analytically that when  $\gamma > \frac{(n-3)}{2(n-2)}$ , the optimal sequence is unique and is obtained by placing the longest job in first position and the rest of the jobs in *SPT* order in positions 2 to  $n$ .

**1. Introduction.** In a recent study [1], the concept of past-sequence-dependent setup times is introduced in the well-known single machine scheduling problem. In their study, the setup time is dependent on the jobs that are already scheduled. The objectives considered in their study are minimizing maximum completion time ( $C_{max}$ ), total completion time ( $TC$ ), total absolute difference in completion times ( $TADC$ ), and a bi-criterion objective function with  $TC$  and  $TADC$ . It is shown in their study that the single machine scheduling problem (with past-sequence-dependent setup times) with the above objectives can be solved in  $O(n \log n)$  time. The learning effect has been included by [2] in the single machine scheduling problem with past-sequence-dependent setup times. In this study [2], a set of objectives including  $C_{max}$ ,  $TC$ ,  $TADC$ , along with minimizing the sum of earliness, tardiness and common due date ( $ETCP$ ) penalties is considered. Polynomial time algorithms are proposed by [2] to obtain the optimal solution for the