International Journal of Innovative
Computing, Information and Control
Volume **5**, Number **9**, September **2009**

ICIC International ©2009 ISSN 1349-4198
pp. **2619–2625**

TIME-OPTIMAL CONTROL OF DISCRETE-TIME LINEAR SYSTEMS WITH STATE AND INPUT TIME-DELAYS

Seyed Mohammad Sadegh Modarres and Seyed Mehdi Karbassi

Faculty of Mathematics
Yazd University
Yazd, 89195-741, Iran
{ smodarres; smkarbassi }@yazduni.ac.ir

Received March 2008; revised September 2008

Abstract. *This paper considers time-optimal control of linear discrete- time delayed systems. An important recent result in the solution of the time-optimal control problem of discrete-time systems with input-delays is extended here to take into account all types of delays in the states, inputs or both. The main result is a parametric expression for the state feedback controller gain matrix with linear parameters which may be used to minimize the norm of the feedback controller gain matrix. A simple algorithm is derived and a numerical example is given to illustrate the effectiveness of our method for systems with different kinds of delays.*
Keywords: Multivariable control systems, Time-delay systems, Eigenvalue assignment, Parameterizations

1. **Introduction.** Time-delays commonly occur in many dynamical systems in the path between system inputs and system outputs. As examples, consider engineering applications where time-delays can include measurement delays, transmission delays and calculation delays. It is essential that system models account for these time-delays in order to predict the true dynamics of the system.

It appears that it was Kurzweil [1] who first constructively established a direct synthesis technique for the time-optimal control of sampled-data systems with input time-delay (sometimes referred to as pure transport delay or transport lag). Koepcke [2] later proposed the method of augmentation of the state vector for time-delayed systems, and since then a number of other investigators [3-7] have used Koepcke's idea in the synthesis of control laws for time-delayed systems. The aim of this paper is to construct state feedback matrix for augmented system so that the closed-loop system has zero eigenvalues and lowest nilpotency index. Accordingly, a synthesis based upon the augmentation of the state vector is presented. Many authors have also considered the robust control problem for time-delay systems (such as Dugard and Verriest, [8] 1998; Mahmoud, [5] 2000). Gu and Niculescu [3] 2003 and Richard [9] 2003 have recently reviewed recent advances concerning time-delay systems.

Linear multivariable discrete-time systems with time-delays fall into three categories. The first category comprises systems in which all the inputs are delayed by the same or different amount to an integer sub-multiple of the time-delay, and are referred to as input-delayed systems; the second category comprises systems in which the states are delayed by the same or different amounts, and are referred to as state-delayed systems. The third category comprises systems in which the states and inputs are delayed by the same or different amounts. The control laws for single and multiple input delayed multivariable systems have been discussed in detail in [1], however, the other two cases