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STABILITY AND HOPF BIFURCATION ANALYSIS IN COUPLED LIMIT CYCLE OSCILLATORS WITH TIME DELAY

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Abstract. *The amplitude death in coupled systems gets great concern. We investigate the stability and Hopf bifurcation at zero equilibrium point. The amplitude death region is obtained. Based on the existence of Hopf bifurcation, the normal form method and center manifold theorem are used to determine the direction of the Hopf bifurcation and the stability of the bifurcating periodic solution. Furthermore, the existence of degenerate double Hopf bifurcation is studied. Quasi-periodicity and chaos are seen at the critical values of degenerate double Hopf bifurcation by numerical simulations. We affirm that chaos really occurs by the largest Liapunov exponent.*

Keywords: Stability, Time delay, Hopf bifurcation, Degenerate double Hopf bifurcation, Chaos

1. **Introduction.** Coupled systems can display abundant dynamic behavior including synchronization, phase locking, bursting [1], Hopf oscillator, chaos [2, 3] and amplitude death [2-4]. Coupled oscillator systems can describe and deal with many biological, chemical, optical and physical problems [5-8]. At present, the dynamics and kinds of collective behavior of coupled oscillator systems become the research hotspots. One of the simplest and earliest of such models is the so-called Kuramoto model [6], which is a mean field model of a collection of phase oscillators, and clearly exhibits such a cooperative phenomenon as spontaneous synchronization of the oscillators beyond a certain coupling strength. Time delay arising from finite propagation speeds of signals between the systems is inevitable. There is an extensive research on the effects of time delay on dynamic systems [9-17]. [18] found some new phenomena in the Kuramoto model such as bistability between synchronized and incoherent states and unsteady solutions with time-dependent order parameters when time delay was introduced. They derived the exact formulas for the stability boundaries of the incoherent and synchronized states. [19] investigated changes of the local Lyapunov stability and Hopf bifurcation caused by time delay for a single limit cycle oscillator. [20, 21] proposed the following coupled limit cycle oscillator model:

$$\begin{aligned}\dot{Z}_1(t) &= (1 + i\omega_1 - |Z_1(t)|^2)Z_1(t) + K[Z_2(t-\tau) - Z_1(t)],\\ \dot{Z}_2(t) &= (1 + i\omega_2 - |Z_2(t)|^2)Z_2(t) + K[Z_1(t-\tau) - Z_2(t)],\end{aligned} \tag{1}$$

where $Z_j(t)$ is the complex amplitude of the jth oscillator. Each oscillator has a stable limit cycle of unit amplitude $|Z_j| = 1$ with angular frequency ω_j. $K \geq 0$ is the coupling strength and $\tau \geq 0$ is a measure of time delay. They showed changes in the stability boundaries of amplitude death, phase locked and incoherent regions using a combination of analytical and numerical methods. In contrast to the corresponding coupled oscillators