THE PORTFOLIO SELECTION PROBLEM WITH RANDOM INTERVAL-VALUED RETURN RATES

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ABSTRACT. The famous mean-variance portfolio selection model introduced by Markowitz in [7] is an important breakthrough in mathematical finance, which deals with uncertainties appearing in financial markets. In real financial market, there may exist two kinds of uncertainties. One is randomness and the other is impreciseness or vagueness. In this paper we study the portfolio selection problem combining randomness with impreciseness by considering asset return rates and risks as random intervals and propose two models. For these models it is important to give a suitable ordering for intervals. Here we use γ -index and satisfactory crisp equivalent system to do it given by Sengupta et al. in [11]. As an application of our models, a numerical example is given whose data comes from a real stock market.

Keywords: Interval-valued random variable, Portfolio selection, $\gamma\text{-index},$ Satisfactory crisp equivalent system

1. Introduction. The mean-variance methodology for the portfolio selection problem, originally proposed by Markowitz in 1952, has been one of the most important research results in modern finance. Following his work, many portfolio selection models have been studied (e.g. [9,10,14,16]). But most of authors only paid more attention to the randomness in economical actions. In practice, however, investors only have partial or imprecise knowledge about markets so that these only random models are not suitable for them to make inferences. In fact, people often use natural language to describe their information. For instance, a risky asset price also can fluctuate quickly due to the market effects. Maybe someone cannot tell you the exact price at some time of future but he can say that 'it will be between \$9.1 and \$9.2' or 'it may be about \$9', i.e. his answer is NOT a precise number but an interval or a natural language description that can be described by a fuzzy number. So there are two kinds of uncertainties in such kind of situation. One is randomness and the other is vagueness or impreciseness so that we need to combine probabilistic tool with other tools to give new models.

Recently, some authors have proposed possibilistic portfolio selection models based on fuzzy theory. In [13], Tanaka *et al.* considered the possibility grades of security data offered by experts, proposed two kinds of portfolio selection models based on possibility distributions rather than traditional probability distributions. Inuiguchi and Tanino dealt with portfolio selection under independent possibilistic information in [5]. Both of work treated the expected return rates of the securities as fuzzy or possibilistic variables instead