International Journal of Innovative Computing, Information and Control
Volume **5**, Number **10(A)**, October **2009**

ICIC International ©2009 ISSN 1349-4198
pp. 3201–3206

NUMERICAL SOLUTIONS OF NEUTRAL FUNCTIONAL DIFFERENTIAL SYSTEMS BY HYBRID OF BLOCK-PULSE FUNCTIONS AND CHEBYSHEV POLYNOMIALS

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Received April 2008; revised September 2008

Abstract. *Using the operational properties of general block-pulse functions and Chebyshev polynomials, the neutral functional differential systems are transformed into a system of algebraic equations. The numerical solutions of the systems are derived. Moreover, applying the results to the linear quadratic optimal control problems, the approximate solutions of optimal control for the neutral functional differential systems are obtained.*
Keywords: Block-pulse functions, Chebyshev polynomials, Neutral functional differential systems

1. Introduction.

The theory of neutral functional differential systems has been developing rapidly. Many authors have given considerable attention to neutral functional differential systems. Many orthogonal functions or polynomials, such as Walsh [1], block-pulse [2], Chebyshev [3], Laguerre [4], Legendre [5], and Fourier [6], were developed for solving various problems of dynamic systems. Motivated by the hybrid functions [8, 9] and Legendre wavelets [10, 11], we solve the neutral functional differential systems by hybrid of general block-pulse functions and Chebyshev polynomials. One of the advantages of this method is that we can divide the integration internal in our control not necessarily to choose K as the power of 2. So the results will model the discontinuities properly. The general operational matrix by hybrid of general block-pulse functions and Chebyshev polynomials is first introduced. We derive the formula of delay functions. Further the neutral functional differential systems are reduced to the algebraic equations. Applying the results to the optimal control problem we obtain the approximate solutions of the optimal control and state as well as the optimal value of the objective functional. Numerical examples are illustrated how to use the algorithms in practice.

2. Preliminaries.

2.1. Definitions.

A set of block-pulse function $b_k(t),\ k = 1, 2, \dots, K$ on the interval $[t_0, t_f)$ are defined as

$$b_k(t) = \begin{cases} 1, & t_{k-1} \le t < t_k, \\ 0, & \text{otherwise} \end{cases}$$

where $t_K = t_f$ and $[t_{k-1}, t_k) \subset [t_0, t_f)$, $k = 1, 2, \dots, K$.

The m order Chebyshev polynomials in the interval $[-1, 1]$ are defined by the following:

$$T_m(t) = \cos(m \arccos(t)) \tag{1}$$