International Journal of Innovative Computing, Information and Control
Volume **5**, Number **5**, May **2009**

ICIC International ©2009 ISSN 1349-4198
pp. 1189–1199

A NOTE ON MULTIPLE LYAPUNOV FUNCTIONS AND STABILITY CONDITION FOR SWITCHED AND HYBRID SYSTEMS

Guisheng Zhai, Isatada Matsune, Joe Imae and Tomoaki Kobayashi

Department of Mechanical Engineering
Osaka Prefecture University
Sakai, Osaka 599-8531, Japan
{ zhai; jimae; kobayasi }@me.osakafu-u.ac.jp

Received April 2008; revised August 2008

Abstract. *In this note, we consider stability condition with multiple Lyapunov functions for switched and hybrid systems. By using the idea of evaluating "the average value of multiple Lyapunov functions" during the activation interval of each subsystem, we propose a new Lyapunov stability condition, which complements the existing stability conditions that evaluate the value of multiple Lyapunov functions at the starting points or the ending points. We present several demonstrative examples, including the case where unstable subsystems are involved, and also apply the stability condition and the switching strategy to the switching control problem for stabilization of nonholonomic systems.*
Keywords: Switched and hybrid systems, Multiple Lyapunov functions, Lyapunov stability, Switching strategy, Average value of multiple Lyapunov functions, Nonholonomic systems

1. Introduction.

In this note, we consider the following *switched and hybrid system*

$$\dot{x}(t) = f_i(x(t)), \ \ i \in \{1, 2, \cdots, N\} \tag{1}$$

where $x(t) \in \Re^n$ is the state, each vector field f_i is globally Lipschitz continuous, N is the number of subsystems, and the i's are picked in such a way that there are finite switches in any finite time interval (so that nonZenoness is guaranteed). Throughout this note, we assume $f_i(0) = 0$ for all i's such that the origin is a common equilibrium point for all subsystems. When switchings occur from one subsystem (vector field) to another subsystem (vector field), state jumps may happen in general.

There has been increasing interest in stability analysis and design for the above switched and hybrid system; see the survey papers [1, 2, 3], the recent books [4, 5, 6] and the references cited therein. Recently, there are various extensions in this area. For example, the extension to the case of including unstable subsystems [7], the case of L_2 gain analysis [8, 9], the case of switched discrete-time systems [9, 10], and the case where both continuous-time and discrete-time subsystems exist [11]. The main motivation of considering switched and hybrid systems is that most practical systems are inherently multimodal in the sense that several dynamical subsystems are required to describe their behaviors which may depend on various environmental factors, and thus switched/hybrid systems composed of several subsystems and effective switching strategy are desired in real applications.

In the literature, there are several existing results using *multiple Lyapunov functions* to show stability for switched and hybrid systems. Ref. [12] considered stability of linear switched and hybrid systems (the function f_i's are linear or affine). Under the assumption that the activation time of each subsystem is bounded above and below by