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SINGLE-MACHINE SCHEDULING WITH AN ACTUAL TIME-DEPENDENT LEARNING CONSIDERATION

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Abstract. *This paper deals with single-machine scheduling problems with an actual time-dependent learning consideration. First, we provide a mathematical description of learning effects in scheduling environment. Then we introduce an actual time-dependent learning model, in which the learning effect is defined a function of the ratio of sum of actual processing times of the jobs previously scheduled to total normal processing time of all jobs. We incorporate it into single-machine scheduling problems and show by examples that the optimal schedule for the classical version of the problem is not optimal in the presence of this new actual time-dependent learning effect for the following objective functions: the makespan, the sum of kth power of the completion times, the total weighted completion times, the maximum lateness and the number of tardy jobs. But for some special cases, we show that the shortest processing time first (SPT) rule, the weighted shortest processing time first (WSPT) rule, the earliest due date (EDD) rule and Moore's Algorithm can also construct an optimal schedule for the problem of minimizing these objective functions, respectively. We also use these rules as heuristics for the general cases and analyze their worst-case error bounds.*

Keywords: Scheduling, Time-dependent, Learning effect, Single-machine, Heuristic algorithm, Performance ratio

1. **Introduction.** In classical scheduling problems, the processing time of a job is assumed to be constant and independent of its position in the processing sequence [1, 2]. However, in many realistic production environments, because firms or employees process a job over and over, they learn how to process it more efficiently. As a result, the processing time of a given job is shorter if it is scheduled later, rather than earlier in the sequence. This phenomenon is known as *learning effect* in the literature [3]. Although learning theory was first applied to industry more than 60 years [4], it has only become a topic in scheduling research in recent years.

Biskup [5] and Cheng and Wang [6] were among the pioneers that brought the concept of learning into the field of scheduling. Biskup [5] assumed that the processing time of a job is a log-linear learning curve. He proved that single-machine scheduling problems to minimize the sum of job flow times and the total deviations of job completion times from a common due date are polynomial solvable. Cheng and Wang [6] considered a single-machine scheduling problem in which the job processing times decrease as a result of learning. They showed that the maximum lateness minimization problem is NP-hard in the strong sense. Then they proposed two heuristics and analyzed their worst-case performance. Later, Lee et al. [7] proposed a heuristic algorithm to solve the total completion time minimization problem in a two-machine flowshop scheduling problem. Chen [8], Mosheiov [9], Lin [10], and Wang and Cheng [11] considered some other