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# POSITIVE SOLUTIONS FOR A NONLINEAR $n$TH-ORDER $m$-POINT BOUNDARY VALUE PROBLEMS WITH COEFFICIENT THAT CHANGES SIGN

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Abstract. *Utilizing the nonlinear alternative theorem, we prove the existence of positive solutions for the nonlinear $n$th order $m$-point boundary value problem with coefficient that changes sign. In order to do so, firstly, we give the associated Green's function and its properties. Then we impose conditions on the nonlinear term $a, f$ and parameter $\lambda$ which ensure the existence of positive solutions. Finally, we give a simple example to illustrate the applications of the obtained result. Our result extends and improves some of the existing literature.*
**Keywords:** Higher-order differential equation, nonlinear alternative theorem, Positive solution

1. **Introduction and Preliminaries.** The multi-point boundary value problems for ordinary differential equations arise in a variety of different areas of applied mathematics and physics. The study of multi-point boundary value problems for linear second order ordinary differential equations was initiated by Il'in and Moiseev [1, 2]. Since then, nonlinear multi-point boundary value problems have been studied by several authors. We refer the reader to [3-11] and references cited therein.

In [12], by using the Leggett-Williams fixed point theorem and the Green's function, Guo et al. studied the existence of multiple positive solutions for the nonlinear $n$th order $m$-point boundary value problem

$$\begin{cases} u^{(n)}(t) + f(t,u) = 0, & t \in (0,1), \\ u(0) = 0, u'(0) = \cdots = u^{(n-2)}(0) = 0, u(1) = \sum\limits_{i=1}^{m-2} k_i u(\xi_i), \end{cases}$$

where $f : [0,1] \times [0,\infty) \to [0,\infty)$ is continuous and $n \geq 2, k_i > 0 (i = 1, 2, \cdots, m-2), 0 = \xi_0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < \xi_{m-1} = 1, 0 < \sum\limits_{i=1}^{m-2} k_i \xi_i^{n-1} < 1$.

In order to apply the concavity of solutions in the proofs, all the above results were done under the assumption the nonlinear term $f$ is nonnegative. For the sign changing nonlinearity $f$, few results were done. Recently, applying the Leray-Schauder degree theory, Liu and Ge [13] establish existence results for positive solutions for the $(n-1, 1)$ three-point boundary value problem

$$\begin{cases} u^{(n)}(t) + \lambda a(t) f(u) = 0, & t \in (0,1), \\ u(0) = \alpha u(\eta), u'(0) = \cdots = u^{(n-2)}(0) = 0, u(1) = \beta u(\eta), \end{cases}$$

where $\eta \in (0,1), \alpha \geq 0, \beta \geq 0$, and $a : (0,1) \to R$ may change sign and $R = (-\infty, +\infty)$. $f(0) > 0, \lambda > 0$ is a parameter.