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OPTIMAL CONTROL COMPUTATION FOR DISCRETE TIME TIME-DELAYED OPTIMAL CONTROL PROBLEM WITH ALL-TIME-STEP INEQUALITY CONSTRAINTS

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Abstract. *In this paper, we consider a class of discrete time optimal control problems with time delay and subject to nonlinear all-time-step inequality constraints on both the state and control. By using a constraint transcription technique in conjunction with a local smoothing method, the problem is approximated by a sequence of discrete time optimal control problems with time delay and subject to nonlinear inequality constraints in canonical form. Rigorous analysis is carried out, showing the convergence of the optimal solutions of the approximate problems to the optimal solution of the original problem. We then move on to consider a general class of discrete time optimal control problem with time delay and subject to nonlinear constraints in canonical form. A computational method is developed based on the sequential quadratic programming (SQP) approximation scheme with active set strategy. It solves the discrete time optimal control problem with time delay and subject to canonical constraints as a nonlinear optimization problem. As an application, we consider a tactical logistic decision analysis problem, which is formulated as a discrete time optimal control problems with time delay and subject to all-time-step inequality constraints. Using the computational method proposed, this practical problem is solved effectively, producing much better results than those obtained in existing literature.*

Keywords: Time delayed system, Discrete time system, Optimal control, All-time-step inequality constraints, Constraint transcription, Tactical logistic

1. **Introduction.** For many natural and man-made systems, inherent delays exist during the transmission of information between different parts of the systems. As a consequence, it gives rise to time delayed systems for which the evolution of current states depends on the past and present values of states and controls. Optimal control of time delayed systems has been an active research area since 1960s. For problems involving continuous time systems with time delay, many papers are now available. See, for example, [1-16]. Amongst these references, several computational methods (see [3-6,8-19]) are suggested. For problems involving discrete time systems with time delay, there are much less papers available in the literature. In [20], Kuhn-Tucker theorem of nonlinear programming (see [22]) is used to derive a discrete maximum principle similar to Pontryagin maximum principle for an optimal discrete time system with a pure delay. However, no efficient computational algorithm is proposed using this discrete maximum principle. In