

## ANALYSIS OF OPTIMAL SITUATION DISTRIBUTIONS IN A SPECIAL $2 \times 2$ BI-MATRIX GAME

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**ABSTRACT.** We shall discuss a  $2 \times 2$  bimatrix game which has exactly one mixed Nash equilibrium in which no strategy is pure. We shall first define the concept of optimal situation distribution that is a correlated equilibrium whose marginal distributions form the unique mixed Nash equilibrium and whose situations with positive probability are strictly pure Nash equilibria. Secondly, we shall give a method to find optimal situation distribution in the game. And finally, we shall research the optimal situation distributions in three classical games, i.e. battle of the sexes, game of chicken, and game of warriors.

**Keywords:**  $2 \times 2$  bimatrix game, Optimal situation distribution, Battle of the sexes, Game of chicken, Game of warriors

**1. Introduction.** Kushida et al. [1,2] showed that two player games in the real world are the most basic, widespread and important. Aumann [3] introduced the concept of correlated equilibrium in non-cooperative games. Aumann [4] proved that the set of correlated equilibria for  $n$ -person games in normal form with a finite number of pure strategies is a compact convex polyhedron that contains the set of Nash equilibria. Kuhn [5], Jansen [6] and Van Damme [7] showed that for a bimatrix game, the set of Nash equilibria is a union of a finite number of maximal Nash subsets, which are convex polytopes. Evangelista and Raghavan [8] showed that, for a bimatrix game, every extreme point of a maximal Nash set is an extreme point of the set of correlated equilibria. Shannon [9] introduced the concept of Shannon entropy to describe uncertainty of a random variable. Shannon [10] regarded uncertainty, or Shannon entropy, of a fixed English letter appearing in an article as a matrix game. Thomas [11] regards that, for a matrix game, a player's mixed strategy can add an uncertainty (In fact, the “uncertainty” is Shannon entropy of the mixed strategy, see Jiang, Zhang, and Ding [12]) which could confuse his opponent. Topsøe [13] and Harremoës, Topsøe [14] researched code games, which are relative to maximum entropy. Quantum game theory touches upon Shannon entropy as well. For example, see Edward [15] and Edward [16]. Neyman and Okada [17] and Neyman and Okada [18] use Shannon entropy to introduce the concept of strategy entropy of a 2-person zero sum repeat game to describe uncertainty of mixed strategy of the game. Jiang, Zhang and Ding [11] researched matrix games with entropy. Jiang and Zhang [19], Jiang [20] and Jiang [21] researched  $n$ -person non-cooperative games with entropy. Jiang [22] studied continuous games with entropy. Jiang [23] systematically studied matrix games with entropy, continuous games with entropy and  $n$ -person non-cooperative games with entropy and their applications.

Our research will start from  $2 \times 2$  bi-matrix games because they are the simplest and the most basic non-cooperative games, such as prisoners' dilemma, battle of the sexes, game of