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# SCHEDULING PROBLEMS WITH EXPONENTIAL LEARNING FUNCTIONS

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Abstract. *In this paper, we consider the scheduling problems with exponential learning effect. By the exponential learning effect, we mean that the processing time of a job is defined by an exponent function of the sum of the normal processing times of the already processed jobs. We consider the following objective functions: the makespan, the total completion time, the sum of the quadratic job completion times, the total weighted completion time and the maximum lateness. For some special single machine problems and flow shop problems, we show that the problems can be solved in polynomial time.*
**Keywords:** Scheduling, Single machine, Flow shop, Learning effect

## 1. Introduction.

In many branches of industry, there arise problems of ordering jobs on machines [1-21]. In classical scheduling problems the processing time of a job is assumed to be a constant. However, in many realistic problems of operations management, both machines and workers can improve their performance by repeating the production operations. Therefore, the actual processing time of a job is shorter if it is scheduled later in a sequence. This phenomenon is known as the "learning effect" in the literature (Badiru [1]). Scheduling problems with learning effects are relatively young but very vivid areas. See Biskup [2] for a comprehensive review of the literature on various models and aspects of scheduling with learning effects.

Biskup [3] assumed that the processing time of a job is a log-linear learning curve, i.e., if job $J_j$ is scheduled in position $r$ in a sequence, its actual processing time is $p_{jr} = p_j r^a$, where $p_j$ is the normal processing time of job $J_j$, $a \leq 0$ is a constant learning effect. He proved that single machine scheduling problems to minimize the sum of job flow times and the total deviations of job completion times from a common due date are polynomial solvable. Wang and Xia [4] and Wang [5] also extended their results to the model: Pegels' learning curve (Pegels [6]), i.e., if job $J_j$ is scheduled in position $r$ in a sequence, its actual processing time is $p_{jr} = p_j[\alpha a^{r-1} + \beta]$, where $\alpha, a$ and $\beta$ are parameters obtained empirically. Koulamas and Kyparisis [7] considered a single machine scheduling problem with the sum-of-job-processing-times-based learning curve, i.e., if job $J_j$ is scheduled in position $r$ in a sequence, its actual processing time is $p_{jr} = p_j \left(1 - \frac{\sum_{i=1}^{r-1} p_{[i]}}{\sum_{i=1}^{n} p_i}\right)^a = p_j \left(\frac{\sum_{i=r}^{n} p_{[i]}}{\sum_{i=1}^{n} p_i}\right)^a$, where $a \geq 1$ is the learning index. They showed that the SPT-sequence is the optimal sequence for the objectives of minimizing the makespan