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COMPARISON OF ALGORITHMS FOR SOLVING A BI-LEVEL TOLL SETTING PROBLEM

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Abstract. *In this paper, we consider the problem of assigning optimal tolls to the arcs of a multi-commodity transportation network. The problem is formulated as a bi-level mathematical program where the upper level is managed by a firm that raises revenues from tolls set on arcs of the network and the lower level is represented by a group of car users traveling along the cheapest paths with respect to a generalized travel cost. The problem can be interpreted as finding an equilibrium among tolls generating high revenues and tolls attracting customers.*

We describe the bi-level programming model and discuss the underlying assumptions. Next, we propose and evaluate four algorithms based on different principles to solve the toll optimization problem. In order to solve this problem efficiently, we first reformulate it as a standard mathematical program and describe a penalty-function algorithm for its solution. The algorithm is well-founded and its convergence is established. We then detail a proposed quasi-Newton-type algorithm, a gradient approximation-based algorithm, and a direct method making use of the Nelder-Mead flexible simplex search. The results of the numerical experiments support the algorithms' robustness.

Keywords: Optimum toll setting problem, Bi-level programming, Penalty function methods, Nelder-Mead algorithm, Gradient approximation method, Quasi-Newton-type method

1. **Introduction.** In many decision processes there exists a hierarchy of decision makers, therefore decisions are to be made at different levels in this hierarchy. One way to handle such hierarchies is to focus on one level and consider the behavior of other levels as assumptions. The research area which focuses on the whole hierarchy structure is called *multi-level programming*. In terms of modeling, the constraint domain associated with

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