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ANALYSIS AND DESIGN OF THE FWL DIGITAL OBSERVER/CONTROLLER IMPLEMENTATIONS: EIGENVALUE SENSITIVITY APPROACH

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Abstract. *In this paper, a methodology using small-gain theorem and eigenvalue sensitivity for analyzing the stability of the closed-loop digital control system subject to finite word length floating point (FWLFP) operations is proposed. The roundoff and computational errors of the observer/controller parameters caused by FWLFP operations are expressed in function of mantissa bit number. First, a sufficient stability criterion in terms of the mantissa bit number for the closed-loop system is derived based on small gain theorem. Then, the magnitude and supplemental angle measures are derived from the sensitivities of the closed-loop system eigenvalues with respect to observer/controller parameters, respectively, in the sense of mixed matrix-2/Frobenius norms. Therefore, an optimal similarity transformation can be obtained from Hermitian solution by considering both the magnitude and supplemental angle measures simultaneously. Based upon the optimal similarity transformation as well as the stability criterion, a minimum mantissa bit number for implementing observers/controllers in FWLFP digital computers can be obtained. The advantages of the proposed methodology are that it can handle the closed-loop systems with complex eigenvalues and give an implementable real-valued optimal similarity transformation by easily algebraic operations. Finally, detailed numerical design processes and simulation results are performed to illustrate the effectiveness of the proposed scheme.*

Keywords: Finite word length, Floating point, Roundoff error, Computational error, Pole sensitivity, Phase sensitivity, Small gain theorem

1. **Introduction.** Computer controls for implementing the stabilizing observers or controllers have been widely used in many industrial products. Since most digital computers use finite word length (FWL) binary number system to tackle any number systems in real world, the assumption that digital computers with infinite precision mathematical operations and memory storages is required. However, FWL effect will cause quantization, rounding, and conversion/computational errors, etc., and exists inevitably for implementing on the observer/controller parameters in digital computers. To date, there are two streams of research in studying implementations in digital computers subject to FWL