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STATE ESTIMATION OF CONSTRAINED NONLINEAR DISCRETE-TIME DYNAMICAL SYSTEMS

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Abstract. *In this paper, a recursive state estimator is developed to handle the problem of state estimation of nonlinear discrete-time dynamical systems when some of the states of these systems are subject to equality or inequality constraints which are due to physical or practical considerations. The system model and the measurements are assumed to be corrupted by zero mean white Gaussian noise. The proposed algorithm, which is based on the multiple projection approach, differentiates between two cases: the case of strictly satisfied constraints and the case when some of the constraints are violated. In the first case, the obtained estimator is the standard extended Kalman filter. However, in the second case, we use the active set method to formulate a set of equality constraints; the number of these constraints equals the number of violated constraints. This set of equality constraints is treated as a new set of noise free measurements. Then, instead of formulating a quadratic optimization problem to be solved in order to satisfy the violated constraints, we use the multiple projection approach to extend the estimator recursively to handle the assumed new set of measurements. The proposed technique is different from other techniques in the sense that it is recursive in addition to the fact that the obtained estimates propagate with the algorithm to the next sampling instant.*

To illustrate the effectiveness and the simplicity of the developed estimation technique, two examples are presented. Simulation results indicate that the proposed technique is simple and effective in estimating the states of nonlinear discrete-time systems while satisfying equality and/or inequality constraints imposed on the states of these systems.
Keywords: State estimation, Linear and nonlinear constraints, Extended Kalman filter

1. **Introduction.** It is well known that the Kalman filter is the optimal minimum variance state estimator for linear dynamical systems with Gaussian noise; and it is the optimal linear estimator for linear dynamical systems with non-Gaussian noise. However, to estimate the states of nonlinear dynamical systems, various modifications to the Kalman filter were proposed. These modifications lead to sub-optimal state estimators. A well known technique used to approximate the optimal state estimator of nonlinear dynamical systems is the extended Kalman filter (EKF).

In many applications of state estimation problems, some equality and/or inequality constraints are imposed on the states of the system; these constraints are due to physical or practical considerations. The standard Kalman filter and the extended Kalman filter do not incorporate such information, and hence they lead to sub-optimal estimators. In these situations, it is necessary to modify the estimators resulting from the standard Kalman filter or the EKF to incorporate such additional information.

Several algorithms were proposed to handle nonlinear state estimation without constraints. These algorithms include the Moving Horizon estimation, the particle filter, the ensemble Kalman filter, the unscented Kalman filter and the extended Kalman filter