International Journal of Innovative
Computing, Information and Control
Volume **6**, Number **10**, October **2010**

 ISSN 1349-4198
pp. 4579–4592

THE CRITICAL CURVE OF THE LONG-TERM RATE OF RETURN WHEN HOLDING RISK ASSETS

Xuefeng Wang, Lei Xu and Qianqing Fang

School of Management
Harbin Institute of Technology
Harbin 150001, P. R. China
wangxuefeng1125@126.com; { xulei523; fqq1016 }@163.com

Received June 2009; revised October 2009

Abstract. *This paper used the Monte Carlo method to randomly simulate the rate of return series, and then studied the characteristics of the long-term investment rate of return and its variance. The computing results showed that, when expressing the long-term rate of return, the Geometrical Brown Motion Model describing the prices of risk assets is very different from the reality. The computing results also showed that, when compared with the short-term rate of return and its variance, both the long-term rate of return and variance decrease with the increasing of investment term; and when the mean of short-term rate of return is a relatively minor positive value, the mean of long-term rate of return might be significantly negative some times. We obtained the critical curve of the rate of return when holding risk assets for a long term through the Monte Carlo method, and deduced several empirical formulas that can exactly express the critical curve by the econometrics. All of these formulas express the relationship between the average rate of return of long-term investment and the short-term rate of return and its variance. This paper also proposed the concept of long-term investment risk premium, and explained the significance of critical curve from the point of investments.*
Keywords: Short-term rate of return, Long-term rate of return, Monte Carlo method, Risk assets, Successive investment

1. **Introduction.** People are familiar with the relation between short-term rate of return and long-term rate of return. As we have known, the long-term rate of return is the geometric mean of all short-term rate of return, and the arithmetic mean of all short-term rate of return is considered as the expectation. In the analysis of investment, people are accustomed to measure the return level by the expectation of short-term rate of return, and measure the uncertainty (risks) of investment returns by the variance of short-term rate of return. However, if investors plan to invest in some risk assets in a relatively long period continuously, it is blindly and error-prone to judge the long-term rate of return and variance of risk assets based on the expectation and variance of their short-term rate of return. As we know, if the short-term rate of return is random, the rate of return of successive investment in a certain period is also random, and there is no simple theoretical formula expressing the relation between the rate of return and variance of short-term investment and that of long-term investment.

For example, if the annual rate of return of some assets obeys normal distribution, its expectation is 10% and its standard deviation is 0.35, people will think that the average annual rate of return when holding this assets for a long period might be close to 10% and just the long-term rate of return has a higher uncertainty without careful calculation. However, the computing results show that the average annual rate of return when holding the assets for a long period is far below 10%. Taking 30 years for example, the mean of