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A NEW ITERATION APPROACH TO SOLVE A CLASS OF FINITE-HORIZON CONTINUOUS-TIME NONAFFINE NONLINEAR ZERO-SUM GAME

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Abstract. *In this paper, a new iteration approach is derived to solve the optimal strategies for quadratic zero-sum game of finite-horizon continuous-time nonaffine nonlinear system. Through iteration algorithm between two sequences which are a sequence of state trajectories of linear quadratic zero-sum games and a sequence of corresponding Riccati differential equations, the optimal strategies for the nonaffine nonlinear zero-sum game are given. Under very mild conditions of local Lipschitz continuity, the convergence of approximating linear time-varying sequences is proved. A numerical example is given to demonstrate the convergence and effectiveness of the proposed approach.*
Keywords: Zero-sum game, Nonaffine nonlinear system, Riccati equation, Approximation theory, Iteration algorithm

1. **Introduction.** Game theory is concerned with the study of decision making in situations where two or more rational opponents are involved under conditions of conflicting interests, which has been widely investigated by many authors [3-8,10,21-24]. The two-player zero-sum game with a quadratic performance index plays an important role in the game theory. One player tries to minimize the performance index while the other tries to maximize it.

Much work has been contributed to this minimax problem under the frameworks of linear quadratic zero-sum games [1,2,18-20,28]. In [1], Al-Tamimi *et al.* applied the heuristic dynamic programming and dual heuristic dynamic programming structures to solve a discrete-time linear quadratic zero-sum game problem in which the state and action spaces are continuous. Then, they designed the optimal strategies for the discrete-time linear quadratic zero-sum game without knowing the system dynamical matrices by the model-free Q-learning approach [2]. A class of continuous-time affine nonlinear quadratic zero-sum game problem was researched by Wei *et al.* in [21]. Abu-Khalaf *et al.* studied the affine nonlinear zero-sum game problem which is converted from solving H_∞ control of nonlinear systems with constrained input system by policy iteration in [16] and used neural networks to solve it in [17]. It is worthy of mentioning that most of the above