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A NOVEL MODAL SERIES REPRESENTATION APPROACH TO SOLVE A CLASS OF NONLINEAR OPTIMAL CONTROL PROBLEMS

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Abstract. *This paper presents a new approach to solve a class of nonlinear optimal control problems which have a quadratic performance index. In this approach, the nonlinear two-point boundary value problem (TPBVP), derived from the Pontryagin's maximum principle, is transformed into a sequence of linear time-invariant TPBVP's. Solving the proposed linear TPBVP sequence in a recursive manner leads to the optimal control law and the optimal trajectory in the form of uniformly convergent series. Hence, to obtain the optimal solution, only the techniques of solving linear ordinary differential equations are employed. In order to use the proposed method in practice, a control design algorithm with low computational complexity and fast convergence rate is presented. Through the finite iterations of algorithm, a suboptimal control law is obtained for the nonlinear optimal control problem. Finally, numerical examples are included to demonstrate efficiency, simplicity and high accuracy of the proposed method.*
Keywords: Nonlinear optimal control problem, Pontryagin's maximum principle, Modal series, Suboptimal control

1. **Introduction.** One of the most active subjects in control theory is the optimal control which has a wide range of practical applications not only in all areas of physics but also in economy, aerospace, chemical engineering, robotic, etc. [1-4]. For linear time-invariant systems, optimal control theory and its application have been developed perfectly [5,6]. However, optimal control of nonlinear systems is much more challenging which has been studied extensively for decades [7,8].

One familiar scheme for the optimal control of nonlinear systems is the state-dependent Riccati equation (SDRE) method [9]. In the SDRE, a direct parameterization is used to transform the nonlinear system into a linear structure with state-dependent coefficients. Then, a matrix Riccati algebraic equation with state-dependent coefficients is solved at each sample state along the trajectory to obtain a nonlinear state feedback control law. Although this method has been widely used in various applications, its major limitation is that it needs solving a sequence of matrix algebraic equations which may take long computing time and large memory space. Moreover, this method produces a suboptimal control law even when the analytic solution of SDRE is available.

Another method, called the approximating sequence of Riccati equations (ASRE), has been introduced in [10]. Although the ASRE is attractive from practical aspects, it suffers from computational complexity due to the need for solving recursively a sequence of linear quadratic time-varying matrix Riccati differential equations.

In order to determine the optimal control law, there is another approach using dynamic programming [11]. This approach leads to the Hamilton-Jacobi-Bellman (HJB) equation