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A REGULATED NEWTON METHOD FOR THE COMPLEMENTARITY PROBLEM OVER EUCLIDEAN JORDAN ALGEBRA WITH SECOND-ORDER CONES

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Abstract. *In this paper, the second-order cone complementarity problem is transformed into a system of algebraic equations by applying the Fischer-Burmeister function. A regulated Newton method is presented to obtain numerical solutions of the problem. By this method, we only need to solve a system of equations at each iteration, without performing any line search. The condition P_0-property is weaker than monotonicity or Cartesian P_0-property which was usually used in existing methods. The validity of the modified technique is shown by illustrative examples and numerical solutions of the problem are calculated with readily computable components. The approximate solutions converge to the exact solution more rapidly than the existing smoothing Newton method.*
Keywords: Complementarity problem, Regulated Newton method, Euclidean Jordan algebra, Second-order cone, FB function

1. **Introduction.** The second-order cone complementarity problem (SOCCP) is to find $(x, y) \in \mathbb{R}^n \times \mathbb{R}^n$ such that

$$x \in K, \quad y \in K, \quad \langle x, y \rangle = 0, \quad y = F(x), \tag{1}$$

where $F : \mathbb{R}^n \to \mathbb{R}^n$ is a continuously differentiable mapping and $K \subseteq \mathbb{R}^n$ is the Cartesian product of second-order cones, that is, $K = K^{n_1} \times K^{n_2} \times \cdots \times K^{n_m}$ with $n_1 + n_2 + \cdots + n_m = n$ and the n_i-dimentional second-order cone $K^{n_i} \subseteq \mathbb{R}^{n_i}$ is defined by

$$K^{n_i} = \left\{ (z_1, z_2^T)^T \in \mathbb{R} \times \mathbb{R}^{n_i - 1} \mid \|z_2\|_2 \leqslant z_1 \right\}.$$

The SOCCP contains a wide class of problems such as the nonlinear complementarity problem (NCP) and the second-order cone programming problem (SOCP). For example, SOCCP with $n_1 = n_2 = \cdots = n_m = 1$ reduces to NCP, and itself is closely related to the KKT optimality conditions for the second-order cone program (SOCP). The SOCCP has wide applications in engineering, economics, management science and other fields; see [1, 2, 3, 4] and references therein. An important reason is that the concept of complementarity is synonymous with the notion of system equilibrium. For instance, the classical Walrasian law of competitive equilibria of exchange economies can be formulated as a nonlinear complementarity problem in the price and excess demand variables. The complementarity condition expresses the fact that the excess demand of a commodity must be zero if its price is positive; similarly, the price of the commodity must be zero if there is positive excess supply. Another example is the physical contact of mechanical structures; here complementarity is between the contact force and the gap (i.e., the distance) between the bodies in contact: the contact force is positive only if there is contact (that is, if the gap is zero).