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# SWARM INTELLIGENCE OPTIMIZED NEURAL NETWORKS FOR SOLVING FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. *In this paper, a swarm intelligence technique, better known as Particle swarm optimization, has been used in solving the fractional differential equations. The approximate mathematical modeling has been done by employing feed-forward artificial neural networks by defining the unsupervised error. The learning of weights for such errors has been carried out by using particle swarm optimization hybridized with simulating annealing algorithms for efficient local search. The design scheme has been successfully applied to solve different problems associated with linear and nonlinear ordinary differential equations of fractional order. The results were compared with available exact solutions, analytic solutions and standard numerical techniques including both deterministic and stochastic approaches. In case of linear ordinary fractional differential equations, relatively more precise solutions were obtained than those of the deterministic numerical methods. Moreover, for complex non-linear fractional differential equations, the technique is still applicable, but with reduced accuracy. The advantages of the proposed scheme are easy implementation, simplicity of concept and broad scope of applications.*
**Keywords:** Computational intelligence, Fractional differential equations, Particle swarm optimization, Neural networks, Numerical computing, Simulating annealing

1. **Introduction.** In the last few decades, fractional differential equations (FDEs) have gained considerable importance due to their varied applications in the fields of applied sciences and engineering [1,2]. The historical survey, theory and applications have been carried out by various writers including Miller and Ross [3], Oldham and Spanier [4] and A. K. Anatoly et al. [5]. The problem to develop numerical solvers for FDEs has attracted many researchers. In this regard, successful advancements have been made in extending the existing classical, as well as, modern numerical solvers. The approximate analytic solutions were derived, and successfully applied to a variety of linear and nonlinear FDEs. Some of the important numerical solvers include Adomian decomposition method [6,7], variational iteration method [8,9], homotopy analysis method [10,11], Taylor collocation method [12], etc. The classical numerical solvers like Grunwald-Letnikov, and Lubich's convolutional quadrature method have also been applied to solve a number of FDEs [13], but with reduced accuracy. Besides this, Diethelm [14] combined the short memory principal with predictor-corrector approach to solve such problems more precisely. Recently, Podlubny [15,16] has provided his famous method of successive approximation in matrix