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UNCONSTRAINED REAL VALUED OPTIMIZATION BASED ON STOCHASTIC DIFFERENTIAL EQUATIONS

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Abstract. *A new approach of finding optimal variables to continuous, real valued functions is outlined. The technique is simple to implement, and does not require multiple instances of the variables or knowledge of the cost function derivative. The variable update is governed by the numerical solution to a specific stochastic differential equation. A number of parameters can be changed to alter the manner in which the variables are updated. Different parameter choices lead to changes in the rate and manner of convergence to the optimal solution. Test functions are used to establish method performance. Results show that the method is competitive with established approaches.*

Keywords: Optimization, Globally convergent, Stochastic differential equation, Evolutionary computing

1. Introduction.

Optimization methods are widely used to estimate a required outcome. The outcome is governed by variables and the ability to compute a set that "best fits" is a requirement of many applications. Applications requiring optimization are vast, ranging from simple curve fitting problems to complex heuristic systems, such as the ones involved in scheduling of tasks or jobs. We present a new paradigm of finding an optimal set of unconstrained variables ($\underset{\sim}{x}$) minimizing a general function f. Symbolically, the problem is stated as:

$$f^* = \min_{\underset{\sim}{x}} f(\underset{\sim}{x}), \quad f \in \mathbb{R}, \quad \underset{\sim}{x} \in \mathbb{R}^L \tag{1}$$

where $f(\underset{\sim}{x}^*) \equiv f^*$ and $\underset{\sim}{x}^*$ is the optimal variable vector of length L that minimizes f. In practice, f has many different forms, and generally, it is termed as the "cost" or the "error" function, as determined by the problem formulation.

Equation (1) can be solved by using deterministic strategies or by randomly searching for the optimal solution. Our work relates to methods that employ random processes to update variables and a number of algorithms have been proposed to solve (1). Existing methods include Simulated Annealing (SA) [1], Genetic Algorithms (GA) [2], Particle Swarm Optimization (PSO) [3], Differential Evolution (DE) [4], Ant Colony Optimization (ACO) [5,6] and others [7]. In these methods, variables evolve over successive iterations, given different random adaptations, mutations, mixing and crossovers. They are, hence, termed Evolutionary Algorithms (EA) [8-11]. Although these methods have been shown to be globally convergent, they do require large numbers of variable instances to be stored at each iteration of the algorithm. This adds complexity to the problem and increases the search space, resulting in optimization methods that are computationally inefficient.

Optimization methods based on the solution to Stochastic Differential Equations (SDE) have been proposed [12,13]. The work was motivated by a quantum mechanics application