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MARKOV SKELETON PROCESS APPROACH TO A CLASS OF PARTIAL DIFFERENTIAL-INTEGRAL EQUATION SYSTEMS ARISING IN OPERATIONS RESEARCH

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Abstract. *A class of partial differential-integral equation systems is derived in operations research including queueing theory, inventory and reliability. The existence and uniqueness for solutions of the class of equation systems have not been investigated yet. In this paper, we present a solution to the class of partial differential-integral equation system via the Markov skeleton process theory and then obtain the existence of the solutions to such equation systems. Furthermore, we prove that the solution is the minimal nonnegative solution of a linear equation system.*
Keywords: Markov skeleton process, Partial differential-integral equation system, Queueing model, Minimal nonnegative solution

1. **Introduction.** In the course of solving stochastic models by the density evolution method, the partial differential-integral equation systems (partial differential equation systems with integral terms) are usually built to characterize the evolutionary behavior of stochastic models. Thus, a large number of partial differential-integral equation systems emerge in the literature of queueing theory, inventory theory and reliability theory.

Example 1.1. *Consider a two-unit parallel reliability model. Suppose the system is composed of two different units and a repairman. The system is in work state if two units work or one works and the other fails. The system is in failure state only if both of the two units fail. If one unit breaks down, the repairman will repair the failed unit immediately. Suppose the lifetime and repairing time are independent random variables with the density functions $f_i(x)$ and $g_i(x)$, $i = 1, 2$, respectively and*

$$F_i(x) = \int_0^x f_i(t)dt, \quad G_i(x) = \int_0^x g_i(t)dt, \quad i = 1, 2.$$

Let

$$L(t) = \begin{cases} 0, & \textit{if two units work at the time } t, \\ 1, & \textit{if unit 1 is being repaired and unit 2 works at the time } t, \\ 2, & \textit{if unit 2 is being repaired and unit 1 works at the time } t, \\ 3, & \textit{if unit 1 is being repaired and unit 2 is to be repaired at the time } t, \\ 4, & \textit{if unit 2 is being repaired and unit 1 is to be repaired at the time } t, \end{cases}$$