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AN INTEGRATED COLUMN GENERATION AND LAGRANGIAN RELAXATION FOR SOLVING FLOWSHOP PROBLEMS TO MINIMIZE THE TOTAL WEIGHTED TARDINESS

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Abstract. *In this paper, we address a new integration of column generation and Lagrangian relaxation for solving flowshop scheduling problems to minimize the total weighted tardiness. In the proposed method, the initial columns are generated by using near-optimal dual variables for linear programming relaxation of Dantzig-Wolfe decomposition derived by the Lagrangian relaxation method. The column generation is executed just after the generation of base columns generated by near-optimal Lagrange multipliers. Computational results demonstrate that the integration of column generation and Lagrangian relaxation can drastically speed up the conventional column generation.*
Keywords: Scheduling, Column generation, Lagrangian relaxation, Flowshop scheduling

1. **Introduction.** Scheduling has become an essential component in recent societies to meet versatile needs [15, 21, 22]. Column generation is an effective computing technique to obtain a tight lower bound for large scale combinatorial optimization problems. The application of the algorithm appears in wide variety of areas such as railway crew scheduling [17], vehicle routing problem [1, 10], ship scheduling [4], patients scheduling in hospital [19] and time tabling problems [27]. It is also benefical to use the algorithm in the branch and price [6] for the exact algorithms. This is due to the fact that a tight lower bound can be obtained from Dantzig-Wolfe reformulation by column generation. Recently, column generation has been successfully applied to production scheduling problems: parallel machine scheduling to minimize the total weighted completion time [8, 31], parallel machine scheduling to minimize earliness and tardiness [9] and flowshop scheduling to minimize the total weighted completion time [16].

It has been well known that the convergence for column generation is extremely slow for large scale problems due to the degeneration in solving the restricted master linear programming problems [20]. This phenomenon is called the tailing-off effect in the column generation. Several improvements for the convergence of column generation have been studied. The oscillation of linear programming dual variables is controlled by the trust region method [20]. The dual cuts are used in stabilizing column generation [3]. The integration of column generation and Lagrangian relaxation (LR) is a promising way to reduce computational efforts for column generation. A combined column generation and Lagrangian relaxation is studied for a single machine scheduling problem with the common due date [30]. They report an improvement of the convergence of column generation by using the lower bound obtained by LR as the criterion of convergence [30]. The reduction