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ADAPTIVE NEURAL CONTROL DESIGN FOR A CLASS OF PERTURBED NONLINEAR TIME-VARYING DELAY SYSTEMS

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Abstract. *In this thesis, Some adaptive neural control design ways are presented for a class of multi-input multi-output (MIMO) nonlinear systems in block-triangular form with disturbance input and state time-varying delay. Neural networks are employed to approximate the unknown continuous functions. By combining the use of a novel quadratic-type Lyapunov-Krasovskii functionals and adaptive NN backstepping, an adaptive neural controller is obtained, which efficiently avoids the controller singularity. The proposed control guarantees that all closed-loop signals remain bounded, while the output tracking error dynamics converges to a neighborhood of the desired trajectories. The feasibility is investigated by a simulation example.*

Keywords: Adaptive neural control, Nonlinear MIMO system, Lyapunov-Krasovskii functional, Backstepping

1. **Introduction.** Time delays are frequently encountered in many real control systems. The existence of the time delays may be the source of instability of serious deterioration in the performance of the closed-loop systems. Meanwhile, perturbations, nonlinearity also exist in most of control systems. Thus, the problem of controlling uncertain time-delay systems has been widely considered in recent years. In [1], T. P. Zhang and S. S. Ge extended the aforementioned result to the adaptive control for a class of MIMO nonlinear state time-varying delay systems. By using Nussbaum type function and Lyapunov-Krasovskii functional, the controller with dead zone was designed. The closed-loop system was proved to be semi-globally uniformly bounded (SGUUB). In [2], the works in Z. Lin and H. Fang (2007), concerned a class of linear input delay systems. By state feedback, the input delay system was transformed into a state delay system. In [3], a class of uncertain time-varying delay system H_∞ control problem is considered, and the corresponding state feedback controller using linear matrix inequalities is proposed.

Neural network control has made great progress in the past decades. Because of their inherent capability for modeling and controlling highly uncertain, nonlinear and complex systems, many neural network control schemes have been introduced to solve the control problem of time delay systems [4-10]. A class of nonlinear state-delay systems is discussed in [4]. Neural network is utilized to estimate the unknown function. By backstepping method, a delay-independent controller is designed. The closed-loop system is proved to be globally uniformly ultimately bounded (GUUB). In [5], a control scheme combined with backstepping, radius basis function (RBF) neural networks and adaptive control is proposed for the stabilization of nonlinear system with input and state delay. By using state transformation the original system is converted to the system without input delay.

In [6], based on a wavelet neural network (WNN) online approximation model, a state feedback adaptive controller is obtained by constructing a novel integral-type Lyapunov-Krasovskii functional, which also efficiently overcomes the controller singularity problem. In [7], based on a neural network (NN) online approximation model, a novel adaptive neural controller is obtained by constructing a novel quadratic-type Lyapunov-Krasovskii functional, which not only efficiently avoids the controller singularity, but also relaxes the restriction on unknown virtual control control coefficients. In [8], by integrating neural-network approximation and the Lyapunov theory into the sliding-mode technique, a neural-network-based sliding-mode control scheme is proposed. In [9], neural networks (NNs) are utilized to approximate and compensate for unknown functions in the system dynamics, including the unknown bounds of the functions of delayed states. The use of a separation technique removes the need for any assumption on the function of delayed states, and allows the handling of multiple delays in each function of delayed states. In [10], the adaptive tracking for a class of nonlinear time delay systems was presented by using a delay state feedback controller. In [11-17,23,24], the problems of the bounded control for delay oscillator uncertain input delay chemistry procedure and the tracking control for nonlinear delay system are considered. In [18], a class of uncertain linear systems with both non-delayed input and delayed input is studied. The controller with delay feedback for the robust stabilization of the system is proposed. The stability criterion of the closed-loop system is derived in terms of LMIS. The main deficiency of [18] is that the system has a non-delayed input so that it cannot be regarded as a pure input delay system. In [19], a class of uncertain linear time-delay systems is considered. By introducing a state predictor, the original system is converted to a normal system without input delay. In [20], neural networks are employed to estimate the unknown continuous functions. The control scheme ensures that the closed-loop system is semi-globally uniformly ultimately bounded (SGUUB). The tracking error is proved to be bounded and ultimately converges to an adequately small compact set. In [22], the state feedback and output feedback adaptive neural network control approaches were presented for a class of strict-feedback discrete time nonlinear systems.

Much work has been done for state-delay nonlinear systems, while less work has been done for state time-varying delay nonlinear systems. However, for the neural-networks control combined with backstepping for the nonlinear system with both state time-varying delay and disturbance input, there is no relevant study.

The above observation motivates the research in this paper. A adaptive neural control design procedure is proposed for state time-varying delay MIMO nonlinear systems in block-triangular form, and a control scheme combined with backstepping, adaptive control and neural networks is presented for the nonlinear system with both state time-varying delay and disturbance input. Radius basis function (RBF) neural network is employed to estimate the unknown continuous function. The proposed control scheme guarantees the boundedness of all the signals in the closed-loop system, and at the same time output tracking is achieved.

2. **Problem Statement.**

Consider the MIMO nonlinear time delay system described by

$$\begin{aligned}
\dot{x}_{j,i_j} &= f_{j,i_j}(\bar{x}_{j,i_j}) + g_{j,i_j}(\bar{x}_{j,i_j})x_{j,i_j+1} + h_{j,i_j}(\bar{x}_{\tau_{j,i_j}}) + \omega_{j,i_j}(t), \\
\dot{x}_{j,m_j} &= f_{j,m_j}(X) + g_{j,m_j}(X)u_j + h_{j,m_j}(X_\tau) + \omega_{j,m_j}(t), \\
y_j &= x_{j,1} \quad j = 1, \cdots, n, i_j = 1, \cdots, m_j - 1,
\end{aligned} \tag{1}$$

where $x_j = [x_{j,1}, \cdots, x_{j,m_j}]^T \in R^{m_j}$ are the delay-free state variables of the jth subsystem, $u_j(t) \in R$ is control input for the first j subsystems, $y = [y_1, \cdots, y_n]^T \in$

R^n is the output, $f_{j,i_j}(.)$, $g_{j,i_j}(.)$ and $h_{j,i_j}(.)$ are unknown smooth nonlinear functions, $\bar{x}_{j,i_j} = [x_{j,1}, \cdots, x_{j,i_j}]^T \in R^{i_j}$ is the vector of delay-free states for the first i_j differential equations of the jth subsystem, $X = [x_1^T, \cdots, x_n^T]^T$ contains all delay-free states, $x_{\tau_{j,i_j}} = x_{j,i_j}(t - \tau_{j,i_j(t)})$ denotes the time-varying delayed state, and $\bar{x}_{\tau_{j,i_j}}$ and X_τ are defined as: $\bar{x}_{\tau_{j,i_j}} = [x_{\tau_{j,1}}, \cdots, x_{\tau_{j,i_j}}]^T$, $X_\tau = [x_{\tau_{1,1}}, \cdots, x_{\tau_{1,n_1}}, \cdots, x_{\tau_{n,1}}, \cdots, x_{\tau_{n,m_n}}]^T$, and $\tau_{j,i_j}(t)$ is the unknown time-varying delay, $|\tau_{j,i_j}(t)| \leq \tau_{j,i_j}$, $|\dot{\tau}_{j,i_j}(t)| \leq \tau_1 < 1$, $\tau_0 = \max\{\tau_{j,i_j} | 1 \leq j \leq n, 1 \leq i_j \leq m_j\}$, $\omega_{j,i_j}(t)$ are the disturbance input for the first j subsystems, $|\omega_{j,i_j}(t)| \leq d_{j,i_j} < 1$ for $t \in [-\tau_0, 0]$. We have $x_{j,i_j}(t) = \beta_{j,i_j}(t)$, $\beta_{j,i_j}(t)$ is smooth and bounded.

The following assumptions and lemmas are made throughout the paper

Assumption 1. The desired trajectories y_{dj} $j = 1, 2, \cdots, n$, and their time derivatives up to the nth order, are continuous and bounded.

Assumption 2. There exist positive functions $Q_{j,l}^{j,i_j}(x_{\tau_{j,l}})$ for $l = 1, 2, \cdots, i_j$. Such that $|h_{j,i_j}(\bar{x}_{\tau_{j,i_j}})| \leq \sum_{l=1}^{i_j} Q_{j,l}^{j,i_j}(x_{\tau_{j,l}})$.

Assumption 3. The signs of $g_{j,i_j}(.)$, for $j = 1, 2, \cdots, n$, $i_j = 1, 2, \cdots, m_j$ are known, and there exist constants g_{j0} and unknown smooth functions $\bar{g}_{j,i_j}(.)$, such that $0 < g_{j0} \leq |g_{j,i_j}(.)| \leq \bar{g}_{j,i_j}(.) < \infty$, without loss of generality. We further assume $g_{j,i_j}(.) > g_{j0} > 0$.

Lemma 2.1. *For any constant $\xi > 0$ and any variable $l \in R$, $\lim_{l \to 0} \tan h^2(l/\xi)/l = 0$.*

Lemma 2.2. *For a given $\varepsilon > 0$ there exists the NN $W^T S(Z)$ can approximate any continuous function $f(Z) \in R^n$, $\Omega_Z \subset R^n$*

$$f(Z) = W^T S(Z) + \theta(Z), \quad |\theta(Z)| \leq \varepsilon, \tag{2}$$

where the input vector $Z \in \Omega_Z \subset R^n$. $W = [w_1, w_2, \cdots, w_l]^T$ is the weight vector; $S(Z) = [s_1(Z), s_2(Z), \cdots, s_l(Z)]^T$, with $l > 1$ being the number of the NN nodes and $s_i(z)$ are defined as $s_i(z) = \exp\left[\frac{-(Z-\mu_i)^T(Z-\mu_i)}{\phi_i^2}\right]$, $i = 1, 2, \cdots, l$, with $\mu_i = [\mu_{i1}, \mu_{i2}, \cdots, \mu_{in}]^T$ the center of the receptive field and ϕ_i the width of the Gaussian function.

3. **Adaptive NN Control Design.** In this section, we develop a novel adaptive NN control design procedure for the jth subsystem. The jth subsystem is composed of m_j design steps. In each step, we employ radial basis function (RBF) NN to approximate the unknown nonlinear function $\bar{f}_{j,i_j}(Z_{j,i_j})$. Thus, define an unknown constant as

$$\delta_j = \frac{1}{g_{j0}} \max\{\|W_{j,i_j}\|^2 : 1 \leq i_j \leq m_j\},$$

where g_{j0} is defined as in Assumption 3, function $\bar{f}_{j,i_j}$ and vector Z_{j,i_j} will be specified in each step. Furthermore, we choose the virtual control law $alpha_{j,i_j}$ and the real control law u_j, respectively, as follows:

$$\alpha_{j,i_j} = -(k_{j,i_j} + 1)z_{j,i_j} - \frac{1}{2a_{j,i_j}^2}\hat{\delta}_j z_{j,i_j} S^T(Z_{j,i_j})S(Z_{j,i_j}), \tag{3}$$

$$u_j = -(k_{j,m_j} + 1)z_{j,m_j} - \frac{1}{2a_{j,m_j}^2}\hat{\delta}_j z_{j,m_j} S^T(Z_{j,m_j})S(Z_{j,m_j}), \tag{4}$$

for $j = 1, 2, \cdots, n$, $i_j = 1, 2, \cdots, m_j$; where $k_{j,i_j} > 0$, and $a_{j,i_j} > 0$ are design parameters, $\hat{\delta}_j$ is the estimation of the unknown constant δ_j, and $S(.)$ is the basis function vector. The error of each step, z_{j,i_j} is defined as:

$$z_{j,i_j} = x_{j,i_j} - \alpha_{j,i_j - 1}, \quad z_{j,1} = x_{j,1} - y_{dj}, \tag{5}$$

for $j = 1, \cdots, n$, $i_j = 2, \cdots, m_j$. The adaptive laws $\dot{\hat{\delta}}_j$ are defined by:

$$\dot{\hat{\delta}}_j = \sum_{i_j=1}^{m_j} \frac{r_j}{2a_{j,i_j}^2} z_{j,i_j}^2 S^T(Z_{j,i_j}) S(Z_{j,i_j}) - b_j \hat{\delta}_j \tag{6}$$

where $r_j > 0$ and $b_j > 0$ are design parameters, when $\omega_{j,i_j}(t) \neq 0$, $1 \leq j \leq n, 1 \leq i_j \leq m_j$: Step $j,1$ $(j = 1, 2, \cdots, n)$ the first step for the jth subsystem. Consider the first equation of the jth subsystem. Consider the Lyapunov function as follows:

$$V_{z_{j,1}} = \frac{1}{2} z_{j,1}^2 + \frac{g_{j0}}{2r_j} \tilde{\delta}^2$$

where $z_{j,1} = x_{j,1} - y_{dj}$, $\tilde{\delta} = \delta_j - \hat{\delta}_j$. The derivative of $V_{z_{j,1}}$ is given by:

$$\dot{V}_{z_{j,1}} = z_{j,1}(f_{j,1} + g_{j,1}\alpha_{j,1} - \dot{y}_{dj} + h_{j,1}(\bar{x}_{\tau_{j,1}}) + \omega_{j,1}(t)) + z_{j,1} g_{j,1} z_{j,2} - \frac{g_{j0}}{r_j} \tilde{\delta}_j \dot{\hat{\delta}}_j. \tag{7}$$

As a result of Assumption 2 and completion of squares, the following inequality is obtained

$$\dot{V}_{z_{j,1}} \leq z_{j,1} \left(f_{j,1} + g_{j,1}\alpha_{j,1} - \dot{y}_{dj} + \frac{1}{2} z_{j,1} + \omega_{j,1}(t) \right) + z_{j,1} g_{j,1} z_{j,2} + \frac{1}{2} \left[Q_{j,1}^{j,1}(x_{\tau_{j,1}}) \right]^2 - \frac{g_{j0}}{r_j} \tilde{\delta}_j \dot{\hat{\delta}}_j. \tag{8}$$

To deal with the delay term in (8). Consider the Lyapunov-Krasovskii functional as follows:

$$V_{u_{j,1}} = \int_{t-\tau_{j,1}(t)}^{t} \frac{1}{2(1-\tau_1)} \left[Q_{j,1}^{j,1}(x_{j,1}(s)) \right]^2 ds.$$

Differentiating $V_{u_{j,1}}$ with respect to time, we obtain

$$\dot{V}_{u_{j,1}} \leq \tfrac{1}{2(1-\tau_1)} \left[Q_{j,1}^{j,1}(x_{j,1}(t)) \right]^2 - \tfrac{1}{2} \left[Q_{j,1}^{j,1}(x_{j,1}(t - \tau_{j,1}(t))) \right]^2. \tag{9}$$

We consider the Lyapunov-Krasovskii functional as follows:

$$V_{j,1} = V_{z_{j,1}} + V_{u_{j,1}}.$$

Differentiating $V_{j,1}$ and using (7), (8) and (9).

$$\dot{V}_{j,1} \leq z_{j,1} \left(\bar{f}_{j,1}(Z_{j,1}) + g_{j,1}\alpha_{j,1} + \omega_{j,1}(t) \right) - \frac{g_{j0}}{r_j} \tilde{\delta}_j \dot{\hat{\delta}}_j + z_{j,1} g_{j,1} z_{j,2} + \left[1 - 2\tan h^2 \left(\frac{z_{j,1}}{\eta_{j,1}} \right) \right] U_{j,1}. \tag{10}$$

$$Z_{j,1} = \left[x_{j,1}, y_{dj}, \dot{y}_{dj}, \hat{\delta}_j \right]^T, \quad U_{j,1} = \frac{1}{2(1-\tau_1)} \left[Q_{j,1}^{j,1}(x_{j,1}) \right]^2,$$

and

$$\bar{f}_{j,1}(Z_{j,1}) = f_{j,1} - \dot{y}_{dj} + \frac{1}{2} z_{j,1} + \frac{2}{z_{j,1}} \tan h^2 \left(\frac{z_{j,1}}{\eta_{j,1}} \right) U_{j,1},$$

with $\eta_{j,1} > 0$.

From Lemma 2.1, we know that the function $\frac{1}{z} \tan h^2 \left(\frac{z}{\eta} \right)$ is defined even at $z = 0$. Thus, the unknown function $\bar{f}_{j,1}(Z_{j,1})$ can be approximated by the NN $W^T S(Z)$. Such that for given $\varepsilon_{j,1} > 0$

$$\bar{f}_{j,1}(Z_{j,1}) = W_{j,1}^T S(Z_{j,1}) + \theta_{j,1}(Z_{j,1}), \quad |\theta_{j,1}(Z_{j,1})| \leq \varepsilon_{j,1}.$$

A straightforward calculation shows that

$$z_{j,1} \bar{f}_{j,1}(Z_{j,1}) \leq \tfrac{1}{2a_{j,1}^2} g_{j0} z_{j,1}^2 \delta_j S^T(Z_{j,1}) S(Z_{j,1}) + \tfrac{1}{2} a_{j,1}^2 + \tfrac{1}{2} g_{j0} z_{j,1}^2 + \tfrac{1}{2} \varepsilon_{j,1}^2 g_{j0}^{-1} \tag{11}$$

From (6), it can be verified that for any initial conditions $\hat{\delta}_j(t_0) \geq 0$, $\hat{\delta}_j(t) > 0$, for all $t > t_0$. Consequently, if follows that

$$z_{j,1}g_{j,1}\alpha_{j,1} \leq \tfrac{1}{2}g_{j0}z_{j,1}^2 + \tfrac{1}{2}d_{j,1}^2 g_{j0}^{-1} \tag{13}$$

Thus, substituting (11), (12) and (13) into (10) result in

$$\begin{aligned}\dot{V}_{j,1} \leq & -k_{j,1}g_{j0}z_{j,1}^2 + \tfrac{1}{2}\left(a_{j,1}^2 + \varepsilon_{j,1}^2 g_{j0}^{-1} + d_{j,1}^2 g_{j0}^{-1}\right) + z_{j,1}g_{j,1}z_{j,2} \\ & + \tfrac{g_{j0}}{r_j}\tilde{\delta}_j\left(\tfrac{r_j}{2a_{j,1}^2}z_{j,1}^2 S^T(Z_{j,1})S(Z_{j,1}) - \dot{\hat{\delta}}_j\right) + \left[1 - 2\tan h^2\left(\tfrac{z_{j,1}}{\eta_{j,1}}\right)\right]U_{j,1}.\end{aligned} \tag{14}$$

Step j, i_j ($j = 1, 2, \cdots, n$, $i_j = 2, \cdots, m_j - 1$) the i_jth step for the jth subsystem. Consider the following Lyapunov-Krasovskii functional:

$$V_{z_{j,i_j}} = \frac{1}{2}z_{j,i_j}^2.$$

The derivative of $V_{z_{j,i_j}}$ is given by

$$\dot{V}_{z_{j,i_j}} = z_{j,i_j}\left(f_{j,i_j} + g_{j,i_j}x_{j,i_j+1} - \dot{\alpha}_{j,i_j-1} + h_{j,i_j}\left(\bar{x}_{\tau_{j,i_j}}\right) + \omega_{j,i_j}(t)\right) \tag{15}$$

Note that $\dot{\alpha}_{j,i_j-1}(Z_{j,i_j-1})$ can be written as:

$$\begin{aligned}\dot{\alpha}_{j,i_j-1} = & \sum_{k=1}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial x_{j,k}}(f_{j,k} + g_{j,k}x_{j,k+1} + \omega_{j,k}) \\ & + \sum_{k=0}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial y_{dj}^{(k)}}y_{dj}^{(k+1)} + \frac{\partial\alpha_{j,i_j-1}}{\partial\hat{\delta}_j}\dot{\hat{\delta}}_j + \sum_{k=1}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial x_{j,k}}h_{j,k}(\bar{x}_{\tau_{j,k}})\end{aligned} \tag{16}$$

As a result of Assumption 2, and completion of squares, the inequality can be rewritten as

$$\begin{aligned}\dot{V}_{z_{j,i_j}} \leq & z_{j,i_j}\Bigg(f_{j,i_j} + g_{j,i_j}x_{j,i_j+1} + \omega_{j,i_j} - \sum_{k=1}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial x_{j,k}}(f_{j,k} + g_{j,k}x_{j,k+1} + \omega_{j,k}) \\ & + \sum_{k=1}^{i_j}\tfrac{1}{2}z_{j,i_j} - \sum_{k=0}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial y_{dj}^{(k)}}y_{dj}^{(k+1)} + \sum_{k=1}^{i_j-1}\sum_{l=1}^{k}\tfrac{1}{2}z_{j,i_j}\left[\frac{\partial\alpha_{j,i_j-1}}{\partial x_{j,k}}\right]^2\Bigg) - \frac{\partial\alpha_{j,i_j-1}}{\partial\hat{\delta}_j}\dot{\hat{\delta}}_j \\ & + \sum_{k=1}^{i_j-1}\sum_{l=1}^{k}\tfrac{1}{2}\left[Q_{j,l}^{j,k}(x_{\tau_{j,l}})\right]^2 + \sum_{k=1}^{i_j}\tfrac{1}{2}\left[Q_{j,k}^{j,i_j}(x_{\tau_{j,k}})\right]^2\end{aligned} \tag{17}$$

To deal with the delay term in (17), we consider the Lyapunov-Krasovskii functional as follows:

$$V_{u_{j,i_j}} = \sum_{k=1}^{i_j}\int_{t-\tau_{j,k}}^{t}\tfrac{1}{2(1-\tau_1)}\left[Q_{j,k}^{j,i_j}(x_{j,k}(s))\right]^2 ds + \sum_{k=1}^{i_j}\sum_{l=1}^{k}\int_{t-\tau_{j,l}}^{t}\tfrac{1}{2(1-\tau_1)}\left[Q_{j,l}^{j,k}(x_{j,l}(s))\right]^2 ds$$

Differentiating $V_{u_{j,i_j}}$, we obtain

$$\begin{aligned}\dot{V}_{u_{j,i_j}} = & \sum_{k=1}^{i_j}\tfrac{1}{2(1-\tau_1)}\left[Q_{j,k}^{j,i_j}(x_{j,k}(t))\right]^2 + \sum_{k=1}^{i_j-1}\sum_{l=1}^{k}\tfrac{1}{2(1-\tau_1)}\left[Q_{j,l}^{j,k}(x_{j,l}(t))\right]^2 \\ & - \sum_{k=1}^{i_j}\tfrac{1}{2(1-\tau_1)}\left[Q_{j,k}^{j,i_j}(x_{\tau_{j,i_j}})\right]^2(1 - \dot{\tau}_{j,k}) - \sum_{k=1}^{i_j-1}\sum_{l=1}^{k}\tfrac{1}{2(1-\tau_1)}\left[Q_{j,l}^{j,k}(x_{\tau_{j,l}})\right]^2(1 - \dot{\tau}_{j,l}) \\ \leq & U_{j,i_j} - \sum_{k=1}^{i_j}\tfrac{1}{2}\left[Q_{j,k}^{j,i_j}(x_{\tau_{j,i_j}})\right]^2 - \sum_{k=1}^{i_j-1}\sum_{l=1}^{k}\tfrac{1}{2}\left[Q_{j,l}^{j,k}(x_{\tau_{j,l}})\right]^2 \\ \leq & z_{j,i_j}\tfrac{2}{z_{j,i_j}}\tan h^2\left(\tfrac{z_{j,i_j}}{\eta_{j,i_j}}\right)U_{j,i_j} + \left[1 - 2\tan h^2\left(\tfrac{z_{j,i_j}}{\eta_{j,i_j}}\right)\right]U_{j,i_j} - \sum_{k=1}^{i_j}\tfrac{1}{2}\left[Q_{j,k}^{j,i_j}(x_{\tau_{j,i_j}})\right]^2 \\ & - \sum_{k=1}^{i_j-1}\sum_{l=1}^{k}\tfrac{1}{2}\left[Q_{j,l}^{j,k}(x_{\tau_{j,l}})\right]^2,\end{aligned} \tag{18}$$

where

$$U_{j,i_j} = \sum_{k=1}^{i_j} \frac{1}{2(1-\tau_1)} \left[Q_{j,k}^{j,i_j}(x_{j,k}(t))\right]^2 + \sum_{k=1}^{i_j-1} \sum_{l=1}^{k} \frac{1}{2(1-\tau_1)} \left[Q_{j,l}^{j,k}(x_{j,l}(t))\right]^2.$$

We consider the Lyapunov-Krasovskii functional as follows:

$$V_{j,i_j} = V_{z_{j,i_j}} + V_{u_{j,i_j}}.$$

Differentiating V_{j,i_j} and using (15), (17) and (18).

$$\begin{aligned} \dot{V}_{j,i_j} \le\; & z_{j,i_j}\left(\varphi_{j,i_j} - \frac{\partial \alpha_{j,i_j-1}}{\partial \hat{\delta}_j}\dot{\hat{\delta}}_j\right) + g_{j,i_j} z_{j,i_j} z_{j,i_j+1} + \left[1 - 2\tan h^2\left(\frac{z_{j,i_j}}{\eta_{j,i_j}}\right)\right] U_{j,i_j} \\ & + z_{j,i_j}(\bar{f}_{j,i_j} + g_{j,i_j}\alpha_{j,i_j+1} + \omega_{j,i_j}), \end{aligned} \tag{19}$$

where

$$\begin{aligned} \bar{f}_{j,i_j} =\; & f_{j,i_j} - \sum_{k=1}^{i_j-1} \frac{\partial \alpha_{j,i_j-1}}{\partial x_{j,k}}(f_{j,k} + g_{j,k}x_{j,k+1} + \omega_{j,k}) + \sum_{k=1}^{i_j} \frac{1}{2} z_{j,i_j} - \sum_{k=0}^{i_j-1} \frac{\partial \alpha_{j,i_j-1}}{\partial y_{dj}^{(k)}} y_{dj}^{(k+1)} \\ & + \sum_{k=1}^{i_j-1} \sum_{l=1}^{k} \frac{1}{2} z_{j,i_j} \left[\frac{\partial \alpha_{j,i_j-1}}{\partial x_{j,k}}\right]^2 + \frac{2}{z_{j,i_j}} \tan h^2\left(\frac{z_{j,i_j}}{\eta_{j,i_j}}\right) U_{j,i_j} - \varphi_{j,i_j}. \end{aligned} \tag{20}$$

The NN $W_{j,i_j}^T S(Z_{j,i_j})$ is used to approximate the unknown function $\bar{f}_{j,i_j}$, such that for given $\varepsilon_{j,i_j} > 0$,

$$\bar{f}_{j,i_j} = W_{j,i_j}^T S(Z_{j,i_j}) + \theta_{j,i_j}(Z_{j,i_j}), \quad \theta_{j,i_j}(Z_{j,i_j})| \le \varepsilon_{j,i_j}.$$

Then, by following a similar line used in the procedure from (11), (12) and (13) to (10) we obtain

$$\begin{aligned} \dot{V}_{j,i_j} \le\; & -k_{j,i_j} g_{j0} z_{j,i_j}^2 + \frac{1}{2}\left(a_{j,i_j}^2 + \varepsilon_{j,i_j}^2 g_{j0}^{-1} + d_{j,i_j}^2 g_{j0}^{-1}\right) + \left[1 - 2\tan h^2\left(\frac{z_{j,i_j}}{\eta_{j,i_j}}\right)\right] U_{j,i_j} \\ & + \frac{g_{j0}}{r_j}\tilde{\delta}_j \frac{r_j}{2a_{j,i_j}^2} z_{j,i_j}^2 S^T(Z_{j,i_j}) S(Z_{j,i_j}) + z_{j,i_j}\left(\varphi_{j,i_j} - \frac{\partial \alpha_{j,i_j-1}}{\partial \hat{\delta}_j}\dot{\hat{\delta}}_j\right) + g_{j,i_j} z_{j,i_j} z_{j,i_j+1} \end{aligned} \tag{21}$$

Step j, m_j $(j = 1, 2, \cdots, n)$ the last step for the jth subsystem. Consider the following Lyapunov-Krasovskii functional:

$$V_{j,i_j} = \frac{1}{2} z_{j,m_j}^2 + V_{u_{j,m_j}},$$

where

$$V_{u_{j,m_j}} = \sum_{j=1}^{n} \sum_{k=1}^{m_j} \int_{t-\tau_{j,k}}^{t} \frac{1}{2(1-\tau_1)} \left[Q_{j,k}^{j,m_j}(x_{j,k}(s))\right]^2 ds + \sum_{k=1}^{m_j-1} \sum_{l=1}^{k} \int_{t-\tau_{j,l}}^{t} \frac{1}{2(1-\tau_1)} \left[Q_{j,l}^{j,k}(x_{j,l}(s))\right]^2 ds.$$

Differentiating $V_{u_{j,m_j}}$, we obtain

$$\begin{aligned} \dot{V}_{u_{j,i_j}} \le\; & z_{j,m_j} \frac{2}{z_{j,m_j}} \tan h^2\left(\frac{z_{j,m_j}}{\eta_{j,m_j}}\right) U_{j,m_j} + \left[1 - 2\tan h^2\left(\frac{z_{j,m_j}}{\eta_{j,m_j}}\right)\right] U_{j,m_j} \\ & - \sum_{j=1}^{n} \sum_{k=1}^{m_j} \frac{1}{2}\left[Q_{j,k}^{j,m_j}(x_{\tau_{j,i_j}})\right]^2 - \sum_{k=1}^{m_j-1} \sum_{l=1}^{k} \frac{1}{2}\left[Q_{j,l}^{j,k}(x_{\tau_{j,l}})\right]^2, \end{aligned}$$

where

$$U_{j,m_j} = \sum_{j=1}^{n} \sum_{k=1}^{m_j} \frac{1}{2(1-\tau_1)} \left[Q_{j,k}^{j,m_j}(x_{j,k}(t))\right]^2 + \sum_{k=1}^{m_j-1} \sum_{l=1}^{k} \frac{1}{2(1-\tau_1)} \left[Q_{j,l}^{j,k}(x_{j,l}(t))\right]^2.$$

Differentiating V_{j,i_j}, similar to (21), we obtain

$$\begin{aligned} \dot{V}_{j,m_j} \le\; & z_{j,m_j}\left(\varphi_{j,m_j} - \frac{\partial \alpha_{j,m_j-1}}{\partial \hat{\delta}_j}\dot{\hat{\delta}}_j\right) + \left[1 - 2\tan h^2\left(\frac{z_{j,m_j}}{\eta_{j,m_j}}\right)\right] U_{j,m_j} \\ & + z_{j,m_j}\left(\bar{f}_{j,m_j} + g_{j,m_j} u_j + \omega_{j,m_j}\right), \end{aligned} \tag{22}$$

where

$$\bar{f}_{j,m_j} = f_{j,m_j} - \sum_{k=1}^{m_j-1} \frac{\partial \alpha_{j,m_j-1}}{\partial x_{j,k}} (f_{j,k} + g_{j,k}x_{j,k+1} + \omega_{j,k}) + \frac{2}{z_{j,m_j}} \tan h^2 \left(\frac{z_{j,m_j}}{\eta_{j,m_j}}\right) U_{j,m_j}$$
$$-\varphi_{j,m_j} + \sum_{k=1}^{m_j} \frac{1}{2} z_{j,m_j} - \sum_{k=0}^{m_j-1} \frac{\partial \alpha_{j,m_j-1}}{\partial y_{dj}^{(k)}} y_{dj}^{(k+1)} + \sum_{k=1}^{m_j-1} \sum_{l=1}^{k} \frac{1}{2} z_{j,m_j} \left[\frac{\partial \alpha_{j,m_j-1}}{\partial x_{j,k}}\right]^2,$$

let $V_{n,m_n} = \sum_{j=1}^{n} \sum_{k=1}^{m_j} V_{j,k}$, then (14), (21) and (22) imply that

$$\begin{aligned}\dot{V}_{n,m_n} \leq & -\sum_{j=1}^{n} \sum_{k=1}^{m_j} k_{j,k} g_{j0} z_{j,k}^2 \sum_{j=1}^{n} \sum_{k=1}^{m_j} \frac{1}{2} \left(a_{j,k}^2 + \varepsilon_{j,k}^2 g_{j0}^{-1} + d_{j,k}^2 g_{j0}^{-1}\right) \\ & + \sum_{j=1}^{n} \frac{g_{j0}}{r_j} \tilde{\delta}_j \left(\sum_{k=1}^{m_j} \frac{r_j}{2a_{j,k}^2} z_{j,k}^2 S^T(Z_{j,k}) S(Z_{j,k}) - \dot{\hat{\delta}}_j\right) \\ & + \sum_{j=1}^{n} \sum_{k=1}^{m_j} \left[1 - 2\tan h^2 \left(\frac{z_{j,k}}{\eta_{j,k}}\right)\right] U_{j,k} + \sum_{j=1}^{n} \sum_{k=2}^{m_j} z_{j,k} \left(\varphi_{j,k} - \frac{\partial \alpha_{j,k-1}}{\partial \hat{\delta}_j} \dot{\hat{\delta}}_j\right),\end{aligned} \tag{23}$$

when $\omega_{j,i_j}(t) = 0$, $1 \leq j \leq n$, $1 \leq i_j \leq m_j$: Step j, 1 ($j = 1, 2, \cdots, n$) the first step for the jth subsystem. Consider the first equation of the jth subsystem. Consider the Lyapunov function as follows:

$$V_{z_{j,1}} = \frac{1}{2} z_{j,1}^2 + \frac{g_{j0}}{2r_j} \tilde{\delta}^2,$$

where $z_{j,1} = x_{j,1} - y_{dj}$, $\tilde{\delta} = \delta_j - \hat{\delta}_j$. The derivative of $V_{z_{j,1}}$ is given by:

$$\dot{V}_{z_{j,1}} = z_{j,1}(f_{j,1} + g_{j,1}\alpha_{j,1} - \dot{y}_{dj} + h_{j,1}(\bar{x}_{\tau_{j,1}})) + z_{j,1} g_{j,1} z_{j,2} - \frac{g_{j0}}{r_j} \tilde{\delta}_j \dot{\hat{\delta}}_j. \tag{24}$$

As a result of Assumption 2 and completion of squares, the following inequality is obtained

$$\dot{V}_{z_{j,1}} \leq z_{j,1} \left(f_{j,1} + g_{j,1}\alpha_{j,1} - \dot{y}_{dj} + \frac{1}{2} z_{j,1}\right) + z_{j,1} g_{j,1} z_{j,2} + \frac{1}{2} \left[Q_{j,1}^{j,1}(x_{\tau_{j,1}})\right]^2 - \frac{g_{j0}}{r_j} \tilde{\delta}_j \dot{\hat{\delta}}_j. \tag{25}$$

To deal with the delay term in (25). Consider the Lyapunov-Krasovskii functional as follows:

$$V_{u_{j,1}} = \int_{t-\tau_{j,1}(t)}^{t} \frac{1}{2(1-\tau_1)} \left[Q_{j,1}^{j,1}(x_{j,1}(s))\right]^2 ds.$$

Differentiating $V_{u_{j,1}}$ with respect to time, we obtain

$$\dot{V}_{u_{j,1}} \leq \frac{1}{2(1-\tau_1)} \left[Q_{j,1}^{j,1}(x_{j,1}(t))\right]^2 - \frac{1}{2} \left[Q_{j,1}^{j,1}(x_{j,1}(t - \tau_{j,1}(t)))\right]^2. \tag{26}$$

We consider the Lyapunov-Krasovskii functional as follows:

$$V_{j,1} = V_{z_{j,1}} + V_{u_{j,1}}.$$

Differentiating $V_{j,1}$ and using (24), (25) and (26).

$$\dot{V}_{j,1} \leq z_{j,1} \left(\bar{f}_{j,1}(Z_{j,1}) + g_{j,1}\alpha_{j,1}\right) - \frac{g_{j0}}{r_j} \tilde{\delta}_j \dot{\hat{\delta}}_j + z_{j,1} g_{j,1} z_{j,2} + \left[1 - 2\tan h^2 \left(\frac{z_{j,1}}{\eta_{j,1}}\right)\right] U_{j,1}, \tag{27}$$

where

$$Z_{j,1} = \left[x_{j,1}, y_{dj}, \dot{y}_{dj}, \hat{\delta}_j\right]^T, \quad U_{j,1} = \frac{1}{2(1-\tau_1)} \left[Q_{j,1}^{j,1}(x_{j,1})\right]^2,$$

and

$$\bar{f}_{j,1}(Z_{j,1}) = f_{j,1} - \dot{y}_{dj} + \frac{1}{2} z_{j,1} + \frac{2}{z_{j,1}} \tan h^2 \left(\frac{z_{j,1}}{\eta_{j,1}}\right) U_{j,1},$$

with $\eta_{j,1} > 0$. From Lemma 2.1, we know that the function $\frac{1}{z} \tan h^2 \left(\frac{z}{\eta}\right)$ is defined even at $z = 0$. Thus, the unknown function $\bar{f}_{j,1}(Z_{j,1})$ can be approximated by the NN $W^T S(Z)$. Such that for given $\varepsilon_{j,1} > 0$

$$\bar{f}_{j,1}(Z_{j,1}) = W_{j,1}^T S(Z_{j,1}) + \theta_{j,1}(Z_{j,1}), \quad |\theta_{j,1}(Z_{j,1})| \leq \varepsilon_{j,1}.$$

A straightforward calculation shows that

$$z_{j,1}\bar{f}_{j,1}(Z_{j,1}) \le \tfrac{1}{2a_{j,1}^2} g_{j0} z_{j,1}^2 \delta_j S^T(Z_{j,1})S(Z_{j,1}) + \tfrac{1}{2}a_{j,1}^2 + \tfrac{1}{2}g_{j0}z_{j,1}^2 + \tfrac{1}{2}\varepsilon_{j,1}^2 g_{j0}^{-1} \tag{28}$$

From (6), it can be verified that for any initial conditions $\hat{\delta}_j(t_0) \ge 0$, $\hat{\delta}_j(t) > 0$, for all $t > t_0$. Consequently, if follows that

$$z_{j,1}g_{j,1}\alpha_{j,1} \le -\tfrac{g_{j0}}{2a_{j,1}^2}\hat{\delta}_j z_{j,1}^2 S^T(Z_{j,1})S(Z_{j,1}) - (k_{j,1}+1)g_{j0}z_{j,1}^2. \tag{29}$$

Thus, substituting (28) and (29) into (27) results in

$$\begin{aligned}\dot{V}_{j,1} \le &-\left(k_{j,1}+\tfrac{1}{2}\right)g_{j0}z_{j,1}^2 + \tfrac{1}{2}\left(a_{j,1}^2 + \varepsilon_{j,1}^2 g_{j0}^{-1}\right) + \left[1 - 2\tan h^2\left(\tfrac{z_{j,1}}{\eta_{j,1}}\right)\right]U_{j,1} \\ &+ \tfrac{g_{j0}}{r_j}\tilde{\delta}_j\left(\tfrac{r_j}{2a_{j,1}^2} z_{j,1}^2 S^T(Z_{j,1})S(Z_{j,1}) - \dot{\hat{\delta}}_j\right) + z_{j,1}g_{j,1}z_{j,2}.\end{aligned} \tag{30}$$

Step j, i_j ($j = 1, 2, \cdots, n$, $i_j = 2, \cdots, m_j - 1$) the i_jth step for the jth subsystem. Consider the following Lyapunov-Krasovskii functional:

$$V_{z_{j,i_j}} = \frac{1}{2}z_{j,i_j}^2.$$

The derivative of $V_{z_{j,i_j}}$ is given by

$$\dot{V}_{z_{j,i_j}} = z_{j,i_j}\left(f_{j,i_j} + g_{j,i_j}x_{j,i_j+1} - \dot{\alpha}_{j,i_j-1} + h_{j,i_j}(\bar{x}_{\tau_{j,i_j}})\right) \tag{31}$$

Note that $\dot{\alpha}_{j,i_j-1}(Z_{j,i_j-1})$ can be written as:

$$\begin{aligned}\dot{\alpha}_{j,i_j-1} = &\sum_{k=1}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial x_{j,k}}(f_{j,k} + g_{j,k}x_{j,k+1}) + \frac{\partial\alpha_{j,i_j-1}}{\partial\hat{\delta}_j}\dot{\hat{\delta}}_j \\ &+ \sum_{k=0}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial y_{dj}^{(k)}}y_{dj}^{(k+1)} + \sum_{k=1}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial x_{j,k}}h_{j,k}(\bar{x}_{\tau_{j,k}})\end{aligned} \tag{32}$$

As a result of Assumption 2, and completion of squares, the inequality can be rewritten as

$$\begin{aligned}\dot{V}_{z_{j,i_j}} \le z_{j,i_j}\Bigg(&f_{j,i_j} + g_{j,i_j}x_{j,i_j+1} - \sum_{k=1}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial x_{j,k}}(f_{j,k} + g_{j,k}x_{j,k+1}) \\ &+ \sum_{k=1}^{i_j}\tfrac{1}{2}z_{j,i_j} - \sum_{k=0}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial y_{dj}^{(k)}}y_{dj}^{(k+1)} + \sum_{k=1}^{i_j}\tfrac{1}{2}\left[Q_{j,k}^{j,i_j}(x_{\tau_{j,k}})\right]^2 \\ &+ \sum_{k=1}^{i_j-1}\sum_{l=1}^{k}\tfrac{1}{2}z_{j,i_j}\left[\frac{\partial\alpha_{j,i_j-1}}{\partial x_{j,k}}\right]^2\Bigg) - \frac{\partial\alpha_{j,i_j-1}}{\partial\hat{\delta}_j}\dot{\hat{\delta}}_j + \sum_{k=1}^{i_j-1}\sum_{l=1}^{k}\tfrac{1}{2}\left[Q_{j,l}^{j,k}(x_{\tau_{j,l}})\right]^2\end{aligned} \tag{33}$$

To deal with the delay term in (17). We consider the Lyapunov-Krasovskii functional as follows:

$$V_{u_{j,i_j}} = \sum_{k=1}^{i_j}\int_{t-\tau_{j,k}}^{t}\tfrac{1}{2(1-\tau_1)}\left[Q_{j,k}^{j,i_j}(x_{j,k}(s))\right]^2 ds + \sum_{k=1}^{i_j}\sum_{l=1}^{k}\int_{t-\tau_{j,l}}^{t}\tfrac{1}{2(1-\tau_1)}\left[Q_{j,l}^{j,k}(x_{j,l}(s))\right]^2 ds$$

Differentiating $V_{u_{j,i_j}}$, we obtain

$$\begin{aligned}\dot{V}_{u_{j,i_j}} \le &\; z_{j,i_j}\tfrac{2}{z_{j,i_j}}\tan h^2\left(\tfrac{z_{j,i_j}}{\eta_{j,i_j}}\right)U_{j,i_j} + \left[1 - 2\tan h^2\left(\tfrac{z_{j,i_j}}{\eta_{j,i_j}}\right)\right]U_{j,i_j} \\ &- \sum_{k=1}^{i_j}\tfrac{1}{2}\left[Q_{j,k}^{j,i_j}(x_{\tau_{j,i_j}})\right]^2 - \sum_{k=1}^{i_j-1}\sum_{l=1}^{k}\tfrac{1}{2}\left[Q_{j,l}^{j,k}(x_{\tau_{j,l}})\right]^2,\end{aligned} \tag{34}$$

where

$$U_{j,i_j} = \sum_{k=1}^{i_j}\frac{1}{2(1-\tau_1)}\left[Q_{j,k}^{j,i_j}(x_{j,k}(t))\right]^2 + \sum_{k=1}^{i_j-1}\sum_{l=1}^{k}\frac{1}{2(1-\tau_1)}\left[Q_{j,l}^{j,k}(x_{j,l}(t))\right]^2.$$

We consider the Lyapunov-Krasovskii functional as follows:

$$V_{j,i_j} = V_{z_{j,i_j}} + V_{u_{j,i_j}}.$$

Differentiating V_{j,i_j} and using (31), (33) and (34).

$$\begin{aligned}\dot{V}_{j,i_j} &= \dot{V}_{z_{j,i_j}} + \dot{V}_{u_{j,i_j}} \\ &\le z_{j,i_j}\left(\varphi_{j,i_j} - \frac{\partial\alpha_{j,i_j-1}}{\partial\hat{\delta}_j}\dot{\hat{\delta}}_j\right) + g_{j,i_j} z_{j,i_j} z_{j,i_j+1} \\ &\quad + \left[1 - 2\tan h^2\left(\frac{z_{j,i_j}}{\eta_{j,i_j}}\right)\right] U_{j,i_j} + z_{j,i_j}\left(\bar{f}_{j,i_j} + g_{j,i_j}\alpha_{j,i_j+1}\right),\end{aligned} \tag{35}$$

where

$$\begin{aligned}\bar{f}_{j,i_j} &= f_{j,i_j} - \sum_{k=1}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial x_{j,k}}(f_{j,k} + g_{j,k}x_{j,k+1}) - \sum_{k=0}^{i_j-1}\frac{\partial\alpha_{j,i_j-1}}{\partial y_{dj}^{(k)}} y_{dj}^{(k+1)} + \sum_{k=1}^{i_j}\frac{1}{2}z_{j,i_j} \\ &\quad + \sum_{k=1}^{i_j-1}\sum_{l=1}^{k}\frac{1}{2}z_{j,i_j}\left[\frac{\partial\alpha_{j,i_j-1}}{\partial x_{j,k}}\right]^2 + \frac{2}{z_{j,i_j}}\tan h^2\left(\frac{z_{j,i_j}}{\eta_{j,i_j}}\right)U_{j,i_j} - \varphi_{j,i_j}.\end{aligned} \tag{36}$$

The NN $W_{j,i_j}^T S(Z_{j,i_j})$ is used to approximate the unknown function $\bar{f}_{j,i_j}$, such that for given $\varepsilon_{j,i_j} > 0$,

$$\bar{f}_{j,i_j} = W_{j,i_j}^T S(Z_{j,i_j}) + \theta_{j,i_j}(Z_{j,i_j}), \quad |\theta_{j,i_j}(Z_{j,i_j})| \le \varepsilon_{j,i_j}.$$

Then, by following a similar line used in the procedure from (28) and (29) to (27), we obtain

$$\begin{aligned}\dot{V}_{j,i_j} &\le -(k_{j,i_j} + \tfrac{1}{2})g_{j0}z_{j,i_j}^2 + \tfrac{1}{2}\left(a_{j,i_j}^2 + \varepsilon_{j,i_j}^2 g_{j0}^{-1}\right) + \frac{g_{j0}}{r_j}\tilde{\delta}_j\frac{r_j}{2a_{j,i_j}^2}z_{j,i_j}^2 S^T(Z_{j,i_j})S(Z_{j,i_j}) \\ &\quad + z_{j,i_j}\left(\varphi_{j,i_j} - \frac{\partial\alpha_{j,i_j-1}}{\partial\hat{\delta}_j}\dot{\hat{\delta}}_j\right) + g_{j,i_j}z_{j,i_j}z_{j,i_j+1} + \left[1 - 2\tan h^2\left(\frac{z_{j,i_j}}{\eta_{j,i_j}}\right)\right]U_{j,i_j}\end{aligned} \tag{37}$$

Step j, m_j ($j = 1, 2, \cdots, n$) the last step for the jth subsystem. Consider the following Lyapunov-Krasovskii functional:

$$V_{j,i_j} = \frac{1}{2}z_{j,m_j}^2 + V_{u_{j,m_j}},$$

where

$$\begin{aligned}V_{u_{j,m_j}} &= \sum_{j=1}^{n}\sum_{k=1}^{m_j}\int_{t-\tau_{j,k}}^{t}\frac{1}{2(1-\tau_1)}\left[Q_{j,k}^{j,m_j}(x_{j,k}(s))\right]^2 ds \\ &\quad + \sum_{k=1}^{m_j-1}\sum_{l=1}^{k}\int_{t-\tau_{j,l}}^{t}\frac{1}{2(1-\tau_1)}\left[Q_{j,l}^{j,k}(x_{j,l}(s))\right]^2 ds\end{aligned}$$

Differentiating $V_{u_{j,m_j}}$, we obtain

$$\begin{aligned}\dot{V}_{u_{j,i_j}} &\le z_{j,m_j}\frac{2}{z_{j,m_j}}\tan h^2\left(\frac{z_{j,m_j}}{\eta_{j,m_j}}\right)U_{j,m_j} + \left[1 - 2\tan h^2\left(\frac{z_{j,m_j}}{\eta_{j,m_j}}\right)\right]U_{j,m_j} \\ &\quad - \sum_{j=1}^{n}\sum_{k=1}^{m_j}\frac{1}{2}\left[Q_{j,k}^{j,m_j}(x_{\tau_{j,i_j}})\right]^2 - \sum_{k=1}^{m_j-1}\sum_{l=1}^{k}\frac{1}{2}\left[Q_{j,l}^{j,k}(x_{\tau_{j,l}})\right]^2,\end{aligned}$$

where

$$\begin{aligned}U_{j,m_j} &= \sum_{j=1}^{n}\sum_{k=1}^{m_j}\frac{1}{2(1-\tau_1)}\left[Q_{j,k}^{j,m_j}(x_{j,k}(t))\right]^2 \\ &\quad + \sum_{k=1}^{m_j-1}\sum_{l=1}^{k}\frac{1}{2(1-\tau_1)}\left[Q_{j,l}^{j,k}(x_{j,l}(t))\right]^2.\end{aligned}$$

Differentiating V_{j,i_j}, similar to (21), we obtain

$$\begin{aligned}\dot{V}_{j,m_j} &\le z_{j,m_j}\left(\varphi_{j,m_j} - \frac{\partial\alpha_{j,m_j-1}}{\partial\hat{\delta}_j}\dot{\hat{\delta}}_j\right) + \left[1 - 2\tan h^2\left(\frac{z_{j,m_j}}{\eta_{j,m_j}}\right)\right]U_{j,m_j} \\ &\quad + z_{j,m_j}\left(\bar{f}_{j,m_j} + g_{j,m_j}u_j\right),\end{aligned} \tag{38}$$

where

$$\bar{f}_{j,m_j} = f_{j,m_j} - \sum_{k=1}^{m_j-1} \frac{\partial \alpha_{j,m_j-1}}{\partial x_{j,k}}(f_{j,k} + g_{j,k}x_{j,k+1}) - \sum_{k=0}^{m_j-1} \frac{\partial \alpha_{j,m_j-1}}{\partial y_{dj}^{(k)}} y_{dj}^{(k+1)} - \varphi_{j,m_j} + \sum_{k=1}^{m_j} \tfrac{1}{2} z_{j,m_j} + \sum_{k=1}^{m_j-1} \sum_{l=1}^{k} \tfrac{1}{2} z_{j,m_j} \left[\frac{\partial \alpha_{j,m_j-1}}{\partial x_{j,k}}\right]^2 + \tfrac{2}{z_{j,m_j}} \tan h^2 \left(\tfrac{z_{j,m_j}}{\eta_{j,m_j}}\right) U_{j,m_j},$$

let $V_{n,m_n} = \sum_{j=1}^{n} \sum_{k=1}^{m_j} V_{j,k}$, then (27), (37) and (38) imply that

$$\begin{aligned} \dot{V}_{n,m_n} \leq & -\sum_{j=1}^{n} \sum_{k=1}^{m_j} \left(k_{j,k} + \tfrac{1}{2}\right) g_{j0} z_{j,k}^2 \sum_{j=1}^{n} \sum_{k=1}^{m_j} \tfrac{1}{2} \left(a_{j,k}^2 + \varepsilon_{j,k}^2 g_{j0}^{-1}\right) \\ & + \sum_{j=1}^{n} \tfrac{g_{j0}}{r_j} \tilde{\delta}_j \left(\sum_{k=1}^{m_j} \tfrac{r_j}{2a_{j,k}^2} z_{j,k}^2 S^T(Z_{j,k}) S(Z_{j,k}) - \dot{\hat{\delta}}_j\right) \\ & + \sum_{j=1}^{n} \sum_{k=1}^{m_j} \left[1 - 2\tan h^2 \left(\tfrac{z_{j,k}}{\eta_{j,k}}\right)\right] U_{j,k} \sum_{j=1}^{n} \sum_{k=2}^{m_j} z_{j,k} \left(\varphi_{j,k} - \tfrac{\partial \alpha_{j,k-1}}{\partial \hat{\delta}_j} \dot{\hat{\delta}}_j\right). \end{aligned} \tag{39}$$

So far, we have completed the completed the control law design.

4. Stability Analysis.

Theorem 4.1. *For system (1), under Assumptions 1, 2 and 3, control law (4) and the NN adaptation law (6), all closed-loop trajectories remain bounded.*

Proof: From (23), (39) it can be seen that the last term may be positive or negative. So we first determine the functions $\varphi_{j,k}$ such that

$$-\sum_{j=1}^{n} \sum_{k=2}^{m_j} z_{j,k} \left(\varphi_{j,k} - \tfrac{\partial \alpha_{j,k-1}}{\partial \hat{\delta}_j} \dot{\hat{\delta}}_j\right) \leq 0. \tag{40}$$

$0 < S^T(.)S(.) < L$. L is the number of neural network weights.

From (6), we obtain

$$\begin{aligned} -\sum_{k=2}^{m_j} z_{j,k} \tfrac{\partial \alpha_{j,k-1}}{\partial \hat{\delta}_j} \dot{\hat{\delta}}_j \leq & \sum_{k=2}^{m_j} z_{j,k} \left(b_j \hat{\delta}_j \tfrac{\partial \alpha_{j,k-1}}{\partial \hat{\delta}_j} - \sum_{l=1}^{k-1} \tfrac{\partial \alpha_{j,k-1}}{\partial \hat{\delta}_j} \tfrac{r_j}{2a_{j,l}^2} z_{j,l}^2 S^T(Z_{j,l}) S(Z_{j,l})\right) \\ & + \sum_{k=2}^{m_j} z_{j,k} \left(\tfrac{r_j L}{2a_{j,k}^2} z_{j,k}^2 \sum_{l=2}^{k} \left|z_{j,l} \tfrac{\partial \alpha_{j,l-1}}{\partial \hat{\delta}_j}\right|\right). \end{aligned}$$

Thus, by choosing $\varphi_{j,k}$ as

$$\varphi_{j,k} = -b_j \hat{\delta}_j \tfrac{\partial \alpha_{j,k-1}}{\partial \hat{\delta}_j} - \tfrac{r_j L}{2a_{j,k}^2} z_{j,k}^2 \sum_{l=2}^{k} \left|z_{j,l} \tfrac{\partial \alpha_{j,l-1}}{\partial \hat{\delta}_j}\right| + \sum_{l=1}^{k-1} \tfrac{\partial \alpha_{j,k-1}}{\partial \hat{\delta}_j} \tfrac{r_j}{2a_{j,l}^2} z_{j,l}^2 S^T(Z_{j,l}) S(Z_{j,l})).$$

(40) holds. Similarly, we obtain

$$\sum_{j=1}^{n} \tfrac{g_{j0}}{r_j} \tilde{\delta}_j \left(\sum_{k=1}^{m_j} \tfrac{r_j}{2a_{j,k}^2} z_{j,k}^2 S^T(Z_{j,k}) S(Z_{j,k}) - \dot{\hat{\delta}}_j\right) \leq \sum_{j=1}^{n} \tfrac{g_{j0}}{2r_j} \left(-\tilde{\delta}_j^2 + \delta_j^2\right). \tag{41}$$

At the present stage, choose Lyapunov functional as $V = V_{n,m_n}$. Then, combining (23), (39) and (40), (41) results in

$$\dot{V}_{n,m_n} \leq -\sum_{j=1}^{n} \sum_{k=1}^{m_j} k_{j,k} g_{j0} z_{j,k}^2 - \sum_{j=1}^{n} \tfrac{g_{j0} b_j}{r_j} \tilde{\delta}_j^2 + \sum_{j=1}^{n} \sum_{k=1}^{m_j} \left[1 - 2\tan h^2 \left(\tfrac{z_{j,k}}{\eta_{j,k}}\right)\right] U_{j,k} + D, \tag{42}$$

where D is as follows:

$$D = \sum_{j=1}^{n} \sum_{k=1}^{m_j} \frac{1}{2} \left(a_{j,k}^2 + \varepsilon_{j,k}^2 g_{j0}^{-1} + d_{j,k}^2 g_{j0}^{-1}\right) + \sum_{j=1}^{n} \frac{g_{j0}}{r_j} \delta_j^2$$

Thus, by (42) the boundedness follows immediately form following the same line used in the proof of References [9]. The proof is thus completed.

5. **Simulation Studies.** When the disturbance input $\omega_{1,1}(t) = \omega_{1,2}(t) \neq 0$, $\omega_{2,1}(t) = \omega_{2,2}(t) \neq 0$. Consider the following nonlinear Systems with time-varying delay and disturbances as follows:

$$\begin{aligned} \dot{x}_{1,1} &= -x_{1,1} + (1+\cos^2(x_{1,1}))x_{1,2} + x^2_{\tau_{1,1}} + \omega_{1,1}(t), \\ \dot{x}_{1,2} &= x_{1,1}x_{1,2} + x_{2,1} + x_{2,2} + (1+0.5\cos^2(x_{2,2}))u_1 + x_{\tau_{1,2}} + \omega_{1,2}(t), \\ \dot{x}_{2,1} &= -x_{2,1} + x_{2,2} + x_{\tau_{2,1}} + \omega_{2,1}(t), \\ \dot{x}_{2,2} &= (x_{1,2}+x_{2,1})x_{2,2} + x_{1,1}u_2 + x_{\tau_{1,1}}x_{\tau_{2,2}} + \omega_{2,2}(t), \end{aligned} \tag{43}$$

where $x_{\tau_{j,i_j}} = x_{j,i_j}(t-\tau_{j,i_j})$, $j=1,2$, $i_j=1,2$, and the time-varying delays are: $\tau_{1,1} = 0.8+0.2\sin(t)$, $\tau_{1,2} = 1+0.5\sin(t)$, $\tau_{2,1} = 0.45+0.05\cos(t)$, $\tau_{2,2} = 2+0.1\cos(t)$. Given the reference output signals as: $y_{d1} = 0.5(\sin(t)+\sin(0.5t))$, $y_{d2} = 0.5\sin(t) + \sin(0.5t)$. We choose the design parameters as: $k_{1,1} = k_{1,2} = k_{2,1} = k_{2,2} = 20$, $a_{1,1} = a_{1,2} = 2, a_{2,1} = a_{2,2} = 1$, $r_1 = r_2 = 400$, $b_1 = b_2 = 0.025$. The disturbance input are chose as: $\omega_{1,1}(t) = \omega_{1,2}(t) = 0.04\sin(2\pi t)$, $\omega_{2,1}(t) = \omega_{2,2}(t) = 0.04\cos(2\pi t)$. For the first subsystem, define the variables $z_{1,1} = x_{1,1} - y_{d1}, z_{1,2} = x_{1,2} - \alpha_{1,1}$. Define the virtual control $\alpha_{1,1} = -(k_{1,1}+1)z_{1,1} - \frac{1}{2a^2_{1,1}}\hat{\delta}_1 z_{1,1}S^T(Z_{1,1})S(Z_{1,1})$. Define the real control law $u_1 = -(k_{1,1}+1)z_{1,1} - \frac{1}{2a^2_{1,1}}\hat{\delta}_1 z_{1,1}S^T(Z_{1,1})S(Z_{1,1})$. For the second subsystem, define the variables $z_{2,1} = x_{2,1} - y_{d2}, z_{2,2} = x_{2,2} - \alpha_{2,1}$. Define the virtual control $\alpha_{2,1} = -(k_{2,1}+1)z_{2,1} - \frac{1}{2a^2_{2,1}}\hat{\delta}_2 z_{2,1}S^T(Z_{2,1})S(Z_{2,1})$. Define the real control law $u_2 = -(k_{2,2}+1)z_{2,2} - \frac{1}{2a^2_{2,2}}\hat{\delta}_2 z_{2,2}S^T(Z_{2,2})S(Z_{2,2})$. Select initial values: $x_{j,i_j}(\vartheta) = 0, -\tau_0 \leq \vartheta \leq 0$, $j=1,2$, $i_j = 1,2$ and $[\hat{\delta}_1(0), \hat{\delta}_2(0)]^T = [0,0]^T$.

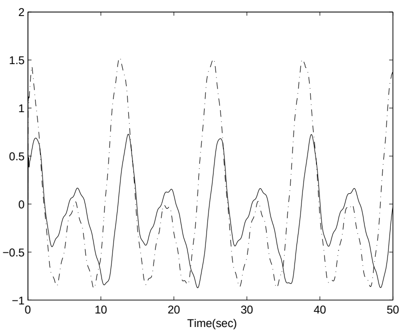

FIGURE 1. System state $x_{1,2}$(“ − ”) and $x_{2,2}$(“ − . − ”)

The result of the control scheme is shown in Figures 1-3. Figure 1 shows the responses of state variables $x_{1,2}$ and $x_{2,2}$. Figure 2 shows the control input signals u_1 and u_2. Figure 3 displays the boundedness of adaptive parameters $\hat{\delta}_1$ and $\hat{\delta}_2$. It can clearly be seen that the proposed controller guarantees the boundedness of all the signals in the closed-loop system.

When the disturbance input $\omega_{1,1}(t) = \omega_{1,2}(t) = 0$, $\omega_{2,1}(t) = \omega_{2,2}(t) = 0$. Consider the following nonlinear systems with time-varying delay as follows:

$$\begin{aligned} \dot{x}_{1,1} &= -x_{1,1} + (1+\cos^2(x_{1,1}))x_{1,2} + x^2_{\tau_{1,1}}, \\ \dot{x}_{1,2} &= x_{1,1}x_{1,2} + x_{2,1} + x_{2,2} + (1+0.5\cos^2(x_{2,2}))u_1 + x_{\tau_{1,2}}, \\ \dot{x}_{2,1} &= -x_{2,1} + x_{2,2} + x_{\tau_{2,1}}, \\ \dot{x}_{2,2} &= (x_{1,2}+x_{2,1})x_{2,2} + x_{1,1}u_2 + x_{\tau_{1,1}}x_{\tau_{2,2}}, \end{aligned} \tag{44}$$

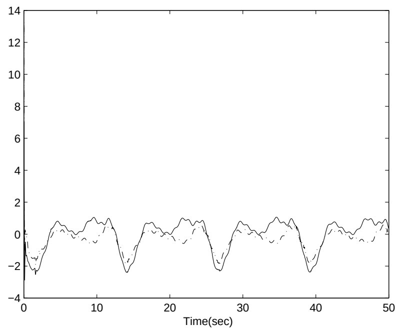

FIGURE 2. The control input u_1(" $-$ ") and u_2(" $-\,.-$ ")

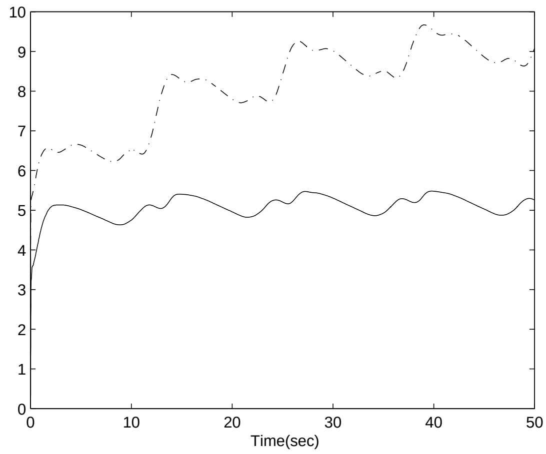

FIGURE 3. The adaptive parameters $\hat{\delta}_1$(" $-$ ") and $\hat{\delta}_2$(" $-\,.-$ ")

where $x_{\tau_{j,i_j}} = x_{j,i_j}(t-\tau_{j,i_j})$, $j = 1,2$, $i_j = 1,2$, and the time-varying delays are: $\tau_{1,1} = 0.8 + 0.2\sin(t)$, $\tau_{1,2} = 1 + 0.5\sin(t)$, $\tau_{2,1} = 0.45 + 0.05\cos(t)$, $\tau_{2,2} = 2 + 0.1\cos(t)$. Given the reference output signals as: $y_{d1} = 0.5(\sin(t) + \sin(0.5t))$, $y_{d2} = 0.5\sin(t) + \sin(0.5t)$. We choose the design parameters as: $k_{1,1} = k_{1,2} = k_{2,1} = k_{2,2} = 20$, $a_{1,1} = a_{1,2} = 2$, $a_{2,1} = a_{2,2} = 1$, $r_1 = r_2 = 400$, $b_1 = b_2 = 0.025$. For the first subsystem, define the variables $z_{1,1} = x_{1,1} - y_{d1}, z_{1,2} = x_{1,2} - \alpha_{1,1}$. Define the virtual control $\alpha_{1,1} = -(k_{1,1}+1)z_{1,1} - \frac{1}{2a_{1,1}^2}\hat{\delta}_1 z_{1,1}S^T(Z_{1,1})S(Z_{1,1})$. Define the real control law $u_1 = -(k_{1,1}+1)z_{1,1} - \frac{1}{2a_{1,1}^2}\hat{\delta}_1 z_{1,1}S^T(Z_{1,1})S(Z_{1,1})$. For the second subsystem, define the variables $z_{2,1} = x_{2,1} - y_{d2}$, $z_{2,2} = x_{2,2} - \alpha_{2,1}$. Define the virtual control $\alpha_{2,1} = -(k_{2,1} + 1)z_{2,1} - \frac{1}{2a_{2,1}^2}\hat{\delta}_2 z_{2,1}S^T(Z_{2,1})S(Z_{2,1})$. Define the real control law $u_2 = -(k_{2,2}+1)z_{2,2} - \frac{1}{2a_{2,2}^2}\hat{\delta}_2 z_{2,2}S^T(Z_{2,2})S(Z_{2,2})$. Select initial values: $x_{j,i_j}(\vartheta) = 0, -\tau_0 \le \vartheta \le 0$, $j = 1,2$, $i_j = 1,2$ and $[\hat{\delta}_1(0), \hat{\delta}_2(0)]^T = [0,0]^T$.

The result of the control scheme is shown in Figures 4-6. Figure 4 shows the responses of state variables $x_{1,2}$ and $x_{2,2}$. Figure 5 shows the control input signals u_1 and u_2. Figure 6 displays the boundedness of adaptive parameters $\hat{\delta}_1$ and $\hat{\delta}_2$. It can clearly be seen that the proposed controller guarantees the boundedness of all the signals in the closed-loop system.

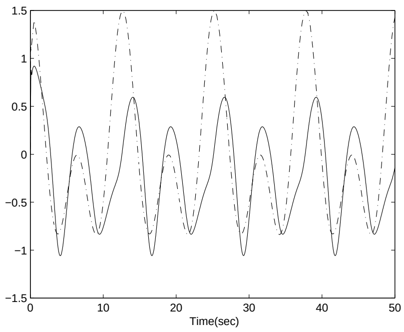

FIGURE 4. System state $x_{1,2}$(" − ") and $x_{2,2}$(" − . − ")

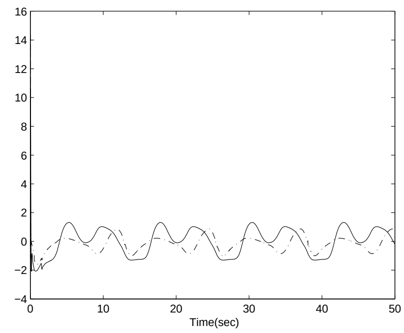

FIGURE 5. The control input u_1(" − ") and u_2(" − . − ")

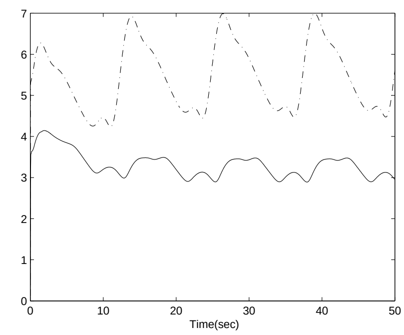

FIGURE 6. The adaptive parameters $\hat{\delta}_1$(" − ") and $\hat{\delta}_2$(" − . − ")

6. **Conclusion.** An adaptive neural network tracking control design scheme has been addressed for a class of nonlinear systems with state time-varying delay and disturbances. RBF neural networks are employed to estimate the unknown continuous functions. The suggested control law guarantees that the tracking errors remain bounded within a neighborhood of the origin. In addition, all other signals in the closed-loop system remain bounded. Simulation has been conducted to show the performance of the proposed approach.

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