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RISK-SENSITIVE REVENUE-SHARING STRATEGY AND SENSITIVITY ANALYSIS IN E-COMMERCE

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Abstract. *In this paper, we develop a new model to describe a dynamic revenue-sharing problem between an online shopping mall and a store in an E-commerce market. We formulate the revenue-sharing problem as a dynamic principal-agent problem, which is then transformed to a risk-sensitive stochastic optimal control problem where the objective of the risk-averse shopping mall is to find a risk-sensitive revenue-sharing strategy and to advise an incentive-compatible effort to the store. Sufficient conditions for the existence of a risk-sensitive revenue-sharing strategy and an incentive-compatible effort are obtained. A numerical example is solved to show the existence of such strategy and its sensitivity to the risk. Moreover, as a comparsion, we also discuss a myopic revene-sharing strategy and find that dynamic revenue-sharing strategy is more effective in expanding the expected profit of the shopping mall.*

Keywords: E-commerce, Revenue-sharing strategy, Dynamic principal-agent problem, Risk-sensitive stochastic control

1. **Introduction.** E-commerce can be viewed as an online two-sided market. In two-sided markets, platforms play important roles. They provide infrastructures and make business rules so that different user groups in the market can conduct their businesses smoothly. A typical example of a platform is Rakuten. Rakuten is the biggest online shopping mall operator in Japan with over 50 million registered users. Rakuten brings stores and customers together to form a two-sided market or a two-sided network.

It is well known that the so-called cross-side network effects exist in a two-sided market, and sellers and buyers in a platform are attracted to each other. Because of the cross-side network effects, increasing the number of users on one side will benefit the users on the platform's other side. In other words, sellers will have more business chances as the number of buyers in a platform increases, and buyers will have more choices and better purchase conditions or better services if more sellers join the platform.

In the early studies on two-sided markets, considerable attention has been paid to how a platform should charge two different user groups in a two-sided market. Theoretical frameworks have been established to explain how the structure of prices is determined. It is well known that the pricing structure in a two-sided market is asymmetric because of different types of users on the two sides [1-4].

E-commerce is a special two-sided market where the online shopping mall only charges stores. The issues faced by a shopping mall in E-commerce are not only to induce participations of stores and customers in the market, but also to make an incentive scheme to stores such that stores can make more efforts to improve the quality of their products or services.

In this paper, we develop a new model to describe a revenue-sharing problem between an online shopping mall and a store. We formulate the problem as a dynamic principal-agent problem where the shopping mall is the principal and the store is the agent. Different from [6], we also assume that the shopping mall is a risk-averse decision maker. Shopping mall's problem is to find a risk-sensitive revenue-sharing strategy and to advise an incentive compatible effort to the store [7]. Sufficient conditions for the existence of a risk-sensitive revenue-sharing strategy and an incentive-compatible effort to the store are obtained. It is believed that the results obtained in this paper can be used as a kind of benchmark for the platform and the seller to determine their contracting condition in practice. A numerical example is solved to show the existence of such strategy and its sensitivity to the risk.

2. **Problem Formulation.** Consider an E-commerce market which consists of one platform (online shopping mall), many sellers (stores) and numerous buyers (customers). Sellers who wish to join the electronic commerce market are required to sign a contract with the platform on the ratio of revenue-sharing. Without loss of generality, we assume that the platform will sign the contract with one seller because homogeneous sellers will be considered in this paper.

Suppose that the initial number of platform's registered members at time $t = 0$ is N_0, which is known by both the platform and the seller. The number of buyers who purchase the seller's products or services, simply the buyers, varies depending on the initial number N_0, the seller's continuous efforts to improve products or services and some other uncertain factors in the market. If the seller makes more efforts to improve the quality of products or services, the number of the buyers will increase. It is assumed in this paper that the number of the buyers can be observed by both the platform and the seller. However, the seller's real effort level is not observable to the platform.

The cumulative number of the buyers at time t is denoted by X_t which evolves according to

$$dX_t = q(a_t)N_0dt + \sigma N_0dZ_t, \tag{1}$$

where $Z = \{Z_t, \mathcal{F}_t; 0 \leq t < \infty\}$ is a standard Brownian motion, σ is a constant, and $\{\mathcal{F}_t; 0 \leq t < \infty\}$ is the filtration determined by $\{X_t; 0 \leq t < \infty\}$. a_t is the seller's choice of effort level and $q(a_t)$ is the quality function of the seller's products or services which is affected by the seller's effort level a_t. The seller's effort range is denoted by $a_t \in [0, \bar{a}]$ where $\bar{a}$ is the upper bound of the effort level. $q(a_t) \in [0, 1]$ is a continuous, strictly increasing and concave function of a_t which is known by the platform. Since $q(a_t)$ can be used to describe the degree of attraction of products or services, it is assumed that all contract members will purchase the seller's products or services if $q(\cdot) = 1$.

For simplicity, the price of the seller's products or services is normalized to one. Hence, X_t is equal to the seller's cumulative sales at time t which is observable to the platform. The seller's sales X_t will be allocated to the seller and to the platform under the conditions of the revenue-sharing contract. Let $\gamma_t \in [0, \infty)$ denote the revenue-sharing strategy at time t made by the platform and the seller. The revenue-sharing strategy γ_t specifies the revenue allocated to the seller. Since the seller's expected sales depend on $q(a_t)$ and the contract number N_0, there is an upper bound to the revenue-sharing strategy γ_t, that is, $\gamma_t \leq \bar{\gamma}_t = N_0$.

Suppose that the seller obtains the utility $u(\gamma_t)$ from the revenue-sharing strategy γ_t, where $u(\gamma_t)$ is a normalized increasing, concave and C^2 function satisfying $u(0) = 0$. On the other hand, the seller incurs the cost of effort $h(a_t)$, measured in the same unit as that of the utility of revenue-sharing strategy, where $h(a_t)$ is a continuous, increasing and convex function of a_t. It is assumed that the platform knows the seller's utility function and the cost function. The risk-averse platform incurs the operational cost βdX_t which depends on the number of the buyers, where $\beta > 0$ is a constant.

For simplicity, it is assumed that both the platform and the seller discount the flow of profit and utility at a common rate r. If the seller chooses an effort level a_t, $0 \le t < \infty$, the seller's total expected utility is given by

$$\mathbb{E}\left[\int_0^\infty e^{-rt}\Big(u(\gamma_t) - h(a_t)\Big)dt\right],$$

and the platform's total expected profit is

$$\begin{aligned}&\mathbb{E}\left[\int_0^\infty e^{-rt}dX_t - \int_0^\infty e^{-rt}\gamma_t dt - \int_0^\infty e^{-rt}\beta dX_t\right]\\ =&\,\mathbb{E}\left[\int_0^\infty e^{-rt}\Big((1-\beta)q(a_t)N_0 - \gamma_t\Big)dt\right].\end{aligned}$$

Since the platform is the risk-averse decision maker, we define the following utility function as the platform's objective function:

$$\mathbb{E}\left\{-\exp\left[-\rho\int_0^\infty e^{-rt}\Big((1-\beta)q(a_t)N_0 - \gamma_t\Big)dt\right] + 1\right\},$$

where ρ is a positive parameter to denote the risk sensitivity of the platform.

2.1. **The platform's problem.** Under the condition of the revenue-sharing contract, the seller will choose an effort a_t to maximize its expected utility. Knowing the behavior of the seller, the platform's problem is to offer a revenue-sharing contract to the seller, which includes an incentive-compatible advice of effort $\{a_t, 0 \le t < \infty\}$ to the seller and a revenue-sharing strategy $\{\gamma_t, 0 \le t < \infty\}$ such that the platform's utility function

$$\mathbb{E}\left\{-\exp\left[-\rho\int_0^\infty e^{-rt}\Big((1-\beta)q(a_t)N_0 - \gamma_t\Big)dt\right] + 1\right\} \tag{2}$$

is maximized. The effort $\{a_t, 0 \le t \le \infty\}$ is referred to as incentive-compatible with respect to the revenue-sharing strategy $\{\gamma_t, 0 \le t \le \infty\}$ if it satisfies

$$a_t \in \arg\max_{\tilde{a}_t}\mathbb{E}\left[\int_0^\infty e^{-rt}\Big(u(\gamma_t) - h(\tilde{a}_t)\Big)dt\right], \tag{3}$$

and

$$\mathbb{E}\left[\int_0^\infty e^{-rt}\Big(u(\gamma_t) - h(a_t)\Big)dt\right] \ge 0. \tag{4}$$

It is obvious that an incentive-compatible effort is relevant to the revenue-sharing strategy γ_t from (3).

2.2. **The seller's continuation value.** In order to make the seller choose a recommended incentive-compatible effort, the platform is required to design a revenue-sharing strategy γ_t which can change the allocation of revenue to the seller according to its effort. Instead of designing a strategy that depends on the sales of the seller, we consider a strategy which depends on the seller's continuation value W_t. The continuation value W_t is the total expected utility received by the seller from time $t \ge 0$ onwards.

Suppose that a revenue-sharing strategy $\gamma = \{\gamma_t\}$ and an effort $a = \{a_t\}$ are given. The seller's continuation value is

$$W_t(\gamma, a) = \mathbb{E}_a\left[\int_t^\infty e^{-r(s-t)}\Big(u(\gamma_s) - h(a_s)\Big)ds\,\Big|\,\mathcal{F}_t\right], \tag{5}$$

where $\mathbb{E}_a$ denotes the expectation under the probability measure $\mathbb{P}_a$ induced by the seller's effort $a = \{a_t\}$. In the platform's revenue-sharing strategy, W_t is the unique state variable that determines how much the seller's allocation of the sales is, what effort the seller is advised to choose, and how W_t itself evolves with the realization of the sales. The platform is required to use W_t as the state feedback to design a revenue-sharing strategy γ_t and a recommended effort a_t to achieve two objectives. First, the seller must have sufficient incentive to choose the recommended effort. Second, the platform's profit is maximized.

It is worth noting that, no matter how much the continuation value W_t is, the platform has the option to stop the revenue-sharing contract with the seller. Suppose that the platform is willing to pay the cancellation cost to the seller. The cancellation cost is determined by the continuation value W_t at the time of cancellation. The platform's profit function at the time of cancellation is $\Omega(W_t) = -\delta\gamma_t$, where $\Omega(0) = 0$ and δ is a constant. Since the seller can choose zero effort after contract cancellation, the seller's continuation value at time t is $W_t = u(\delta\gamma_t)$.

If the seller's continuation value W_t is extremely high, the platform will cancel the contract with the seller. The reason is that the continuation value W_t will increase as the allocation of the sales to the seller increases. However, if the allocation to the seller is too high, the allocation to the platform will be below the operational cost incurred by the platform. Therefore, there must exist a continuation value $W^\sharp > 0$ such that the platform is willing to pay the cancellation cost $\Omega(W^\sharp)$ to stop the contract.

3. **Risk-Sensitive Revenue-Sharing Strategy.** In this section, we will derive the optimal solution to the problem formulated in the above section. First, as a preliminary result, we give the following proposition, which is proved formally in [6] to describe the evolution of the seller's continuation value W_t.

Proposition 3.1. *Suppose that a revenue-sharing strategy $\gamma = \{\gamma_t\}$ and an effort $a = \{a_t\}$ after time $t > 0$ are given. Then, there exists a $\mathcal{F}_t$-progressively measurable process Y_t such that the seller's continuation value $W_t(\gamma, a)$ defined by (5) can be described by the stochastic differential equation*

$$dW_t(\gamma, a) = \big[rW_t(\gamma, a) - u(\gamma_t) + h(a_t)\big]dt + \sigma N_0 Y_t dZ_t. \tag{6}$$

Second, we give the following proposition, which is proved formally in [6] too, to describe the incentive-compatibility condition on the seller's effort.

Proposition 3.2. *Suppose that Y_t is a progressively measurable process defined by Proposition 3.1. Then, the seller's effort a_t is optimal if and only if*

$$a_t \in \arg\max_{\tilde{a}_t \in [0,\bar{a}]} Y_t q(\tilde{a}_t)N_0 - h(\tilde{a}_t), \quad 0 \le t < \infty \tag{7}$$

almost everywhere.

From Proposition 3.2, it is shown that Y_t is the function of the seller's incentive compatible effort a_t, that is,

$$Y_t = \frac{h'(a_t)}{q'(a_t)N_0} = y(a_t) > 0. \tag{8}$$

$y(a_t)$ is an increasing function of a_t. Since Y_t of (6) represents the volatility of the seller's continuation value $W_t(\gamma, a)$, the seller's risk will increase as effort increases.

3.1. **The risk-sensitive stochastic control problem.** Making use of the monotonicity of a logarithmic function, we know that maximizing the utility function (2) is equivalent to the problem of maximizing the following objective function

$$J(W) = -\rho^{-1}\ln\Big(-\mathbb{E}\Big\{-\exp\Big[-\rho\int_t^\infty e^{-r(s-t)}\Big((1-\beta)q(a_s)N_0-\gamma_s\Big)ds\Big]+1\Big\}+1\Big). \quad (9)$$

Suppose that the evolution of the seller's continuation value W_t is known. The platform's control problem to find the optimal revenue-sharing strategy γ_t and the recommended effort a_t, which satisfies the incentive compatibility condition, can be formulated as a risk-sensitive stochastic control problem:

$$\Pi(W) = \max_{a,\gamma} J(W) = -\rho^{-1}\ln\Big(-\psi(W)+1\Big) \quad (10)$$

subject to

$$dW_t = \big[rW_t - u(\gamma_t) + h(a_t)\big]dt + \sigma N_0 y(a_t)dZ_t, \quad (11)$$

where

$$\psi(W) = \max_{\gamma,a}\mathbb{E}\Big\{-\exp\Big[-\rho\int_t^\infty e^{-r(s-t)}\Big((1-\beta)q(a_s)N_0-\gamma_s\Big)ds\Big]+1\Big\}. \quad (12)$$

Furthermore, defining $\Psi(W) = \psi(W) - 1$, we have

$$\Psi(W) = \max_{\gamma,a}\mathbb{E}\Big\{-\exp\Big[-\rho\int_t^\infty e^{-r(s-t)}\Big((1-\beta)q(a_s)N_0-\gamma_s\Big)ds\Big]\Big\}. \quad (13)$$

Therefore, the problem formulated by (10), (11) is equivalent to the problem of maximizing (13) subject to (11). This problem is solved by using dynamic programming, and the Hamilton-Jacobi-Bellman (HJB) equation is obtained below,

$$\max_{a,\gamma}\Big(rW - u(\gamma) + h(a)\Big)\Psi'(W) - \rho\Big((1-\beta)q(a)N_0-\gamma\Big)\Psi(W) + \frac{1}{2}\sigma^2N_0^2y(a)^2\Psi''(W) = 0. \quad (14)$$

Using the transformations $\Psi'(W) = -\rho\Pi'(W)\Psi(W)$ and $\Psi''(W) = -\rho\Pi''(W)\Psi(W) - \rho^2(\Pi'(W))^2\Psi(W)$, we arrive at the HJB equation

$$\max_{a,\gamma}\Big(rW - u(\gamma) + h(a)\Big)\Pi'(W) + \Big((1-\beta)q(a)N_0-\gamma\Big) + \frac{1}{2}\rho\sigma^2N_0^2y(a)^2(\Pi'(W))^2 + \frac{1}{2}\sigma^2N_0^2y(a)^2\Pi''(W) = 0 \quad (15)$$

from (14). The HJB Equation (15) is solved under the initial condition

$$\Pi(0) = 0, \quad (16)$$

and the final conditions [1]

$$\Pi(W^\sharp) = -\Omega(W^\sharp), \quad \Pi'(W^\sharp) = -\Omega'(W^\sharp), \quad (17)$$

at a time τ, where $t=\tau$ is the time point when the platform cancels the contract with the seller and $\Omega(\cdot)$ is the platform's value function when the seller chooses zero effort.

[1] $\Pi(W^\sharp) = -\Psi(W^\sharp)$ is called the value-matching condition and $\Pi'(W^\sharp) = -\Psi'(W^\sharp)$ is called the smooth-pasting condition.

3.2. **Solutions of risk-sensitive stochastic control problem.** Suppose that the solution $\Pi(W)$ to (15) exists. We have the following proposition, which is proved formally in Appendix A.

Proposition 3.3. *Suppose that* $\Pi(W)$ *satisfies the HJB Equation (15) with respect to* $W_t \in [0, W^\sharp]$ *in* $t \in [0, \tau]$*, the initial condition (16) and the final conditions (17) at* $t = \tau$*. If* a_t *and* γ_t *are the seller's recommended effort and the platform's revenue-sharing strategy which maximize the left-hand side of (15), then* a_t *and* γ_t *are the solutions of the risk-sensitive stochastic control problem formulated in Section 3.1.*

From Proposition 3.3, the optimal recommended effort $a(W_t)$ is obtained as the function of W_t by maximizing

$$\max_a\Big(h(a)\Pi'(W) + \frac{1}{2}\rho\sigma^2 N_0^2 y(a)^2(\Pi'(W))^2 + \frac{1}{2}\sigma^2 N_0^2 y(a)^2\Pi''(W) + (1-\beta)q(a)N_0\Big) \quad (18)$$

where $(1-\beta)q(a)N_0$ is the revenue flow, $-h(a)\Pi'(W)$ is the effort compensation to the seller, and $-\frac{1}{2}\rho\sigma^2 N_0^2 y(a)^2(\Pi'(W))^2 - \frac{1}{2}\sigma^2 N_0^2 y(a)^2\Pi''(W)$ is the risk premium paid to the seller in an uncertain business environment.

Similarly, the optimal revenue-sharing strategy is obtained by maximizing

$$\max_\gamma(-\gamma - u(\gamma)\Pi'(W)). \quad (19)$$

From the first-order condition $\Pi'(W) = -1/u'(\gamma)$, $\gamma(W_t)$ is obtained as the function of the continuation value W_t. $-\Pi'(W)$ represents the platform's marginal decrease in the value function with respect to the continuation value. $1/u'(\gamma)$ $(= d\gamma/du(\gamma))$ represents the platform's marginal revenue share with respect to the seller's utility. Moreover, when $W \le W^*$ where W^* is a point such that $\Pi'(W^*) = 0$, since $u(\gamma) \ge 0$ and $\Pi'(W > 0) \ge 0$, $\gamma = 0$ from (19).

The solution $\Pi(W)$ of the HJB Equation (15) can be obtained through numerical computation. In order to show the existence of the solution of (15) and the existence of a corresponding risk-sensitive revenue-sharing strategy and a recommended effort, an illustrative example is solved. The functions and the parameters appeared in the problem formulation are defined as follows:

$$q(a) = a, \quad u(\gamma) = \sqrt{\gamma}, \quad h(a) = 0.5a^2 + 0.5a$$

and

$$N_0 = 1, \quad r = 0.1, \quad \rho = 0.1, \quad \sigma = 1, \quad \beta = 0.1, \quad \delta = 235.$$

The platform's optimal revenue-sharing strategy can be obtained from $\gamma = \Pi'(W)^2/4$, and the seller's optimal recommended effort is

$$a = -0.9/(\Pi'(W) + \rho(\Pi'(W))^2 + \Pi''(W)) - 0.5.$$

The numerical results of the platform's value function, the optimal revenue-sharing strategy and the optimal recommended effort are shown in Figure 1. Moreover, $W^* = 1.256$, $\Pi(W^*) = 1.1170811$.

4. Discussion.

4.1. **Sensitivity analysis of platform's strategies.** Since ρ denotes the risk sensitivity of the platform, the higher the value of ρ is, the more averse the platform is to the risk. In this section, we will analyze how the value of ρ affects the various strategies obtained above by numerical simulation.

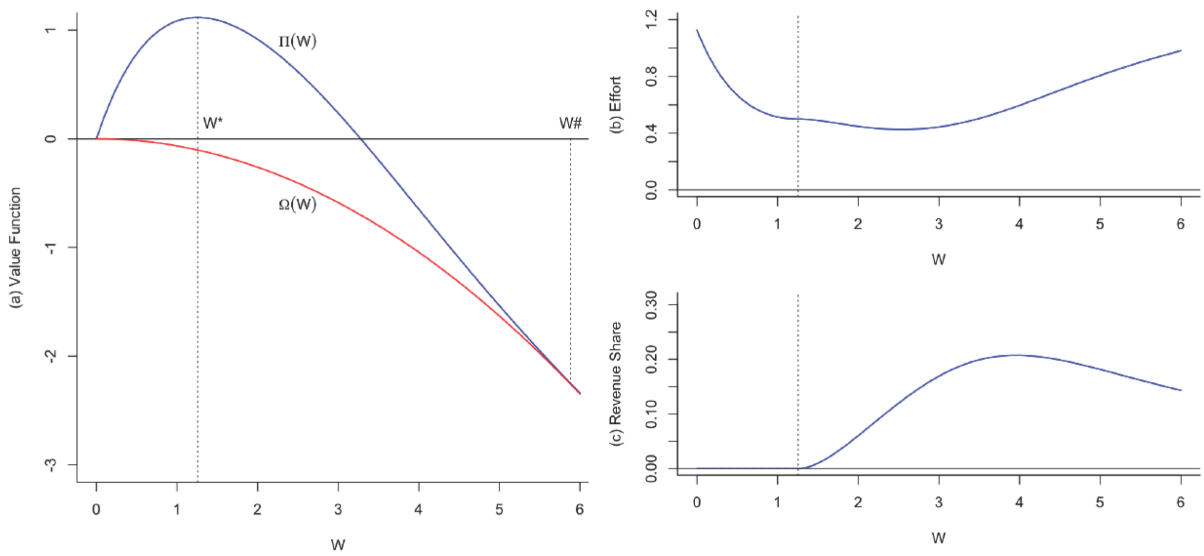

FIGURE 1. (a) Platform's value function, (b) effort, (c) risk-sensitive revenue-sharing strategy

From (18), it is shown that, when $W = W^*$,

$$\max_a \Big(\frac{1}{2}\sigma^2 N_0^2 y(a)^2 \Pi''(W^*) + (1-\beta)q(a)N_0\Big)$$

is independent of ρ, where W^* is the seller's continuation value when $\Pi'(W^*) = 0$. Therefore, the platform's attitude to the risk will not influence the seller's effort level at the point of $W = W^*$. Obviously, W^* will be different as the change of ρ.

It is still not clear how the different ρ will affect the value function $\Pi(W)$. In general, if a decision maker is more risk-averse, he/she might not pursue a bigger profit because the utility will change only a little even if the profit increases a lot. In other words, a relatively smaller profit will lead to the same satisfaction as that of a bigger profit if a decision maker is more risk-averse. Hence, it is estimated that a more risk-averse platform with less profit may have the same satisfaction as that of a less risk-averse platform with bigger profit. However, if the risk is transferable, the estimation above may not be correct. In the following, we will discuss the issues using the numerical example above.

Figure 2 shows curves of $\Pi(W)$, $a(W)$ and $\gamma(W)$ when $\rho = 0.05$, 0.1 and 0.2, respectively. It is found from Figure 2 that the results are different from the estimation above. The higher the value of ρ is, the bigger the value of $\Pi(W^*)$, $0 \le W^* \le W^\sharp$ is. Moreover, the higher the value of ρ is, the higher the value of $\Pi'(0)$ and the value of W^* are. These results can be explained as follows.

First, the higher $\Pi'(0)$ means a higher initial effort $a(W(0)) = a(0)$. In fact, since $\gamma(0) = 0$ at $t = 0$, $\Pi'(0)$ depends on $a(0)$ from the definition of Π. The simulation Figure 2 shows the same results. The higher the value of ρ is, the values of $a(0)$ and W^* become higher.

Now, the risk-averse platform may expect a higher W^* to the seller, which are of great advantages in excluding weak sellers and postponing the allocation of revenue to the seller. It is required that the seller makes much more effort at the beginning if a higher W^* is expected. That also means the increase of the volatility of the seller's continuation value, that is, the seller's risk (see (6)).

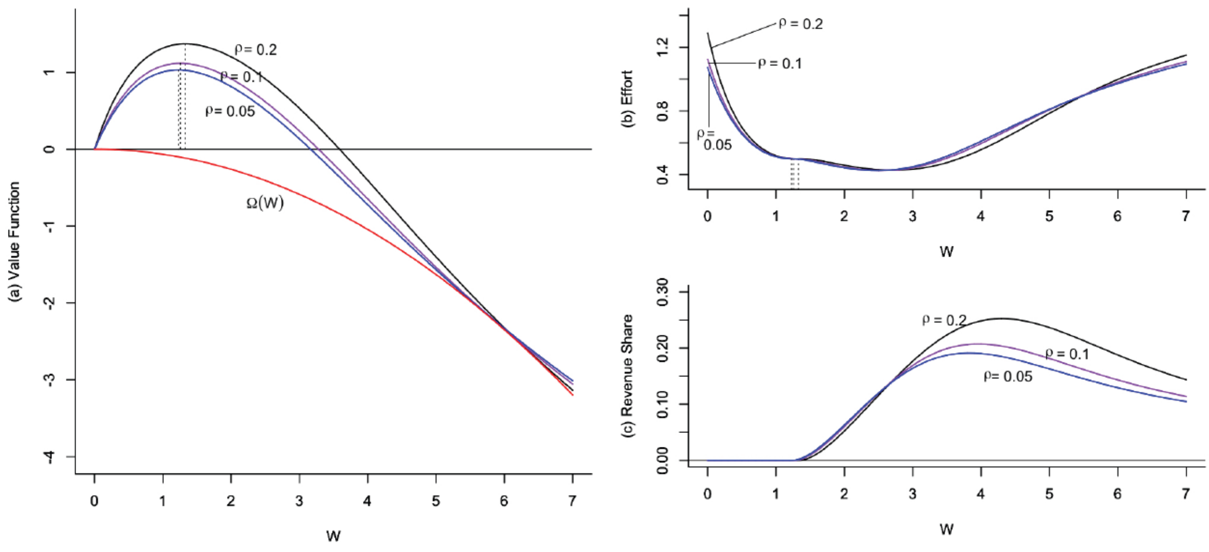

Figure 2. Results of sensitivity analysis

As the result, the risk-averse platform will expect a higher W^* to the seller, which will transfer the risk from the platform to the seller. If W^* is high, the seller is required to make much more effort at the beginning which leads to a high $\Pi'(0)$ and a high $\Pi(W)$.

4.2. **Myopic revenue-sharing strategy.** What is the effect on revenue of the platform and effort of the seller when revenue-sharing strategy is determined myopically, i.e., allocation γ_t is determined with respect to sales X_t only at time t?

There are at least two methods of allocation that the platform can select. One method is to determine revenue-sharing strategy γ_t with respect to the continuation value, taking into account the future expected revenue and seller's risk, as we have demonstrated before. Another method is to myopically determine revenue-sharing strategy γ_t only through the revenue at time t, $X(t)$. There are pros and cons to both methods. The former allows endowment of incentive for the seller to put in greater effort such as long-term expansion (as well as payment of risk premium for stochastic demand on future revenue), but computation of expected revenue is rather complicated. On the other hand, the latter is simple to calculate but promotes the seller to focus on short-term sales increase and also does not take into account the randomness exogenous to the seller's effort. In this section, we will analyze the implication of choosing such myopic revenue-sharing scheme over the expected revenue scheme as described by this paper.

Similar to the problem formulation, we assume that one platform in an E-commerce market will sign the revenue-sharing contract with one seller myopically under the condition that γ.

Firstly, consider the case where uncertainty does not exist in the sales X_t, that is,

$$dX_t = q(a_t)N_0 dt. \tag{20}$$

Since the allocation to the seller is $\gamma q(a_t)N_0$, the utility of the seller is $u(\gamma q(a_t)N_0)$. Moreover, the seller incurs the cost $h(a_t)$. Thus, the seller's myopic optimal effort level with respect to the given γ is

$$a_t^m = \arg\max_{\tilde{a}_t} \Big[u(\gamma q(\tilde{a}_t)N_0) - h(\tilde{a}_t) \Big]. \tag{21}$$

The optimal effort level $a_t^m(\gamma)$ is obtained as the function of γ from the first-order condition $du(\gamma q(a_t)N_0)/da_t - h'(a_t) = 0$, where, $a_t^m(\gamma) = a^m(\gamma)$ is constant. Applying this to the numerical example in Section 3.2, we find that $a^m(\gamma)$ satisfies $\gamma = 4a^m(\gamma)\left(a^m(\gamma)+0.5\right)^2$, $0 \le a^m(\gamma) \le \overline{a}$. Thus, the allocation to the seller at any time is $4\left(a^m(\gamma)\right)^2\left(a^m(\gamma)+0.5\right)^2$.

Since the platform's myopic expected profit for an arbitrary $\tilde{\gamma}$ is

$$\begin{aligned}\Pi^m &= \mathbb{E}\left[\int_0^\infty e^{-rt}\big((1-\beta)q\left(a^m(\tilde{\gamma})\right)N_0 - \tilde{\gamma}q\left(a^m(\tilde{\gamma})\right)N_0\big)dt\right]\\ &= \frac{1}{r}\Big[(1-\beta)q\left(a^m(\tilde{\gamma})\right)N(0) - \tilde{\gamma}q\left(a^m(\tilde{\gamma})\right)N_0\Big], \qquad (22)\end{aligned}$$

$$\Pi^m = \frac{1}{0.1}\left[0.9a^m(\tilde{\gamma}) - 4\left(a^m(\tilde{\gamma})\right)^2\left(a^m(\tilde{\gamma})+0.5\right)^2\right] \qquad (23)$$

when it is applied to the numerical example. Therefore,

$$a^{m*}(\gamma^*) = 0.1872, \quad \gamma^* = 0.3536, \quad \Pi^{m*} = 1.0228.$$

Compared with the results using the dynamic revenue-sharing strategy in Section 3.2, we find that both the effort level and the platform's expected profit are decreasing.

Furthermore, since the seller's sales is uncertain, the risk-premium should also be added when considering the revenue-sharing strategy. Suppose that the seller's risk-averse level is $\lambda = \left|\frac{u''(\cdot)}{u'(\cdot)}\right|$. The risk premium is

$$\gamma_\lambda q(a)N_0 = \frac{1}{2}\lambda\sigma^2 q(a)N_0. \qquad (24)$$

Including the risk-premium to the seller, the platform's myopic expected profit becomes

$$\Pi^{m\lambda} = \frac{1}{r}\left[(1-\beta)q\left(a^m(\gamma)\right)N(0) - \gamma q\left(a^M(\gamma)\right)N_0 - \frac{1}{2}\lambda\sigma^2 q(a)N_0\right]. \qquad (25)$$

Applying this into the numerical example in Section 3.2, we find that

$$\Pi^{m\lambda} = \frac{1}{0.1}\left[0.9a^m(\tilde{\gamma}) - 4\left(a^m(\tilde{\gamma})\right)^2\left(a^m(\tilde{\gamma})+0.5\right)^2 - 0.25\right], \qquad (26)$$

and

$$\Pi^{m\lambda*} = -1.477,$$

which means that the platform's profit is decreasing if the risk-premium is paid. However, if the platform does not pay the risk-premium to the seller, the seller will not have the incentive to improve its effort, which will also affect sales in a long-term perspective.

5. **Conclusion.** In this paper, we have considered the risk-sensitive revenue-sharing problem between a risk-averse platform and a seller in E-commerce. We have formulated the problem as a dynamic principal-agent problem, and then transformed it to a risk-sensitive stochastic control problem where the objective of the platform is to find a risk-sensitive revenue-sharing strategy and to advise an incentive-compatible effort to the seller. Sufficient conditions for the existence of a risk-sensitive revenue-sharing strategy and an incentive-compatible effort are obtained. A numerical example is solved to show the existence of the strategy and the effort, and their sensitivities to the risk.

It is believed that the results obtained in this paper can be used as a kind of benchmark for the platform and the seller to determine their contracting condition in practice. In particular, in markets such as electronic music market or electronic book market where buyers purchase products repeatedly, platforms's dynamic incentive strategy seems necessary which can enforce seller's efforts to improve the product quality. Results in Section 4.2 show that the myopic revenue-sharing strategy will deteriorate the expected profit of the platform compared with the dynamic revenue-sharing strategy. The platform's dynamic revenue-sharing strategy is more effective in giving incentive to the seller for its effort and in expanding the expected profit. In this paper, we have assumed that there exists a monopolistic platform and homogeneous sellers in the market. Further researches are under way to expand the model to the cases of competitive platforms and sellers with different cost structures.

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Appendix A. **Proof of Proposition 3.3.** Suppose that γ_t^*, a_t^* are the solution of (12). We have

$$
\begin{aligned}
\psi(W(t)) = \max_{\gamma,a} \mathbb{E}\Bigg\{ &-\exp\left[-\rho\int_t^{t+\Delta t} e^{-r(s-t)}\Big((1-\beta)q(a_s)N_0-\gamma_s\Big)ds\right] \\
&\cdot \exp\left[-\rho\int_{t+\Delta t}^{\infty} e^{-r(s-t-\Delta t)}\Big((1-\beta)q(a_s)N_0-\gamma_s\Big)ds\right]+1\Bigg\} \\
= \max_{\gamma,a} \mathbb{E}\Bigg\{ &-\exp\left[-\rho\int_t^{t+\Delta t} e^{-r(s-t)}\Big((1-\beta)q(a_s)N_0-\gamma_s\Big)ds\right] \\
&\cdot\Big(-\psi(W(t+\Delta t))+1\Big)+1\Bigg\} \\
= \max_{\gamma,a} \mathbb{E}\Bigg\{ &\left[-1+\rho e^{-r\Delta t}\Big((1-\beta)q(a_t)N_0-\gamma_t\Big)\Delta t\right] \\
&\cdot\Big(-\psi(W(t))-\Delta\psi(W(t))+1\Big)+1\Bigg\} \\
= \max_{\gamma,a} \mathbb{E}\Bigg\{ &\psi(W(t))+\Delta\psi(W(t))-\rho\Big((1-\beta)q(a_t)N_0-\gamma_t\Big)\psi(W(t))\Delta t \\
&+\rho r\Big((1-\beta)q(a_t)N_0-\gamma_t\Big)\psi(W(t))(\Delta t)^2 \\
&-\rho\Big((1-\beta)q(a_t)N_0-\gamma_t)\Big)\Delta\psi(W(t))\Delta t \\
&+\rho r\Big((1-\beta)q(a_t)N_0-\gamma_t)\Big)\Delta\psi(W(t))(\Delta t)^2
\end{aligned}
$$

$$- \rho\Big((1-\beta)q(a_t)N_0 - \gamma_t)\Big)\Delta t$$
$$+ \rho r\Big((1-\beta)q(a_t)N_0 - \gamma_t)\Big)(\Delta t)^2\Bigg\}.$$

Let $\Delta t \to 0$, then $(\Delta t)^2$ and $\Delta\psi(W(t))\Delta t$ converge to 0 as Δt goes to 0. Thus,

$$\max_{\gamma,a} \mathbb{E}\Big\{d\psi(W(t)) - \rho\Big((1-\beta)q(a_t)N_0 - \gamma_t\Big)(\psi(W(t)) - 1)dt\Big\} = 0.$$

Furthermore, using Ito lemma, we have

$$\begin{aligned} d\psi(W(t)) = &\left\{\big[rW(t) - u(\gamma_t) + h(a_t)\big] \cdot \psi'(W(t)) + \frac{1}{2}\sigma^2 N_0^2 y(a_t)^2 \psi''(W(t))\right\}dt \\ &+ \sigma N_0 y(a_t)\psi'(W(t))dZ(t). \end{aligned}$$

Therefore, we arrive at the HJB equation

$$\begin{aligned} \max_{\gamma,a}\big[rW(t) - u(\gamma_t) + h(a_t)\big]\psi'(W(t)) &+ \frac{1}{2}\sigma^2 N_0^2 y(a_t)^2 \psi''(W(t)) \\ &- \rho\Big((1-\beta)q(a_t)N_0 - \gamma_t\Big) \cdot (\psi(W(t)) - 1) = 0. \end{aligned}$$

Letting $\Psi(W(t)) = \psi(W(t)) - 1$, $\Psi'(W(t)) = \psi'(W(t))$ and $\Psi''(W(t)) = \psi''(W(t))$ gives

$$\begin{aligned} \max_{\gamma,a}\big[rW(t) - u(\gamma_t) + h(a_t)\big]\Psi'(W(t)) &+ \frac{1}{2}\sigma^2 N_0^2 y(a_t)^2 \Psi''(W(t)) \\ &- \rho\Big((1-\beta)q(a_t)N_0 - \gamma_t\Big)\Psi(W(t)) = 0. \end{aligned}$$

Therefore, γ_t^*, a_t^* can be obtained through the solutions of (14), which constitute the risk-sensitive revenue-sharing strategy and the incentive-compatible effort. (Q.E.D)