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PRICING AND INSOLVENCY RISK EVALUATING OF EMBEDDED OPTIONS IN UNIVERSAL INSURANCE CONSIDERING MORTALITY RATE

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Abstract. *Focusing on universal life insurance, this paper calculates level premium and constructs the basic model of assets and liabilities considering mortality rate. The model contains interest rate guarantees, surrender options, and mortality rate to fully reflect the reality of prevailing insurance policies. Based on the pricing of universal life insurance, the paper also establishes the insolvency model. Finally, Monte Carlo simulations are conducted to provide numerical results and sensitivity analysis. In order to show the advantages of our model against those without consideration of mortality, we also conducted a mortality scenario test by introducing a mortality modifying factor. Results show that the two embedded derivatives, interest rate guarantee and cancellable option, account for a large proportion in universal insurance's value. This finding may suggest the necessity to consider the interest rate guarantee and cancellable option while assessing the value of the universal insurance, in order to correctly evaluate the solvency of the insurance companies. Results also show that the mortality effect is significant on both embedded option and insolvency risk, which indicates a great necessity of considering mortality in valuation of universal insurance.*

Keywords: Universal life insurance, Mortality rate, Insolvency risk, Embedded option

1. **Introduction.** Recently, great attention has been paid to the options embedded in insurance contracts, especially in those nontraditional contracts which account for a significant part of insurance market, i.e., participating insurance contracts, universal life insurance and unit-linked insurance. Embedded or implicit options often provide policyholder the right to make a change on the time or amount of claims. Nevertheless, traditional approaches in actuarial science usually underestimate the value of these options (see [1]). Consequently, this could cause two kinds of risks to insurance company. First, underestimation of these options for a long while could leave insurance companies in a terrible operational condition. Second, embedded options tend to increase the contingent liability of insurance companies. Without enough liability reserves, insurance companies could easily get trapped in a solvency crisis or even go bankrupt as economy becomes dramatically unstable.

In 2006, when the subprime crisis took place, a great many insurance companies suffered from disastrous impact, which mainly laid in the risk of insolvency. Losses caused by subprime derivatives not only created great liquidity risks but also jeopardized the confidence of policyholders on insurance companies' solvency, which would lead to large-scale surrender and eventually destroy the least hope for insurance companies to go through crisis. For that reason, we should all keep alert of insufficient pricing of insurance contracts with implicit options and its serious consequences. Despite their importance, there have not been much literature on the pricing issue of embedded options and their influence on insolvency. In general, embedded options can be categorized as two different types: one offers the right to make a change on the time or amount of settlement, and the other provides a guaranteed minimum interest rate or return. As mentioned above, traditional approaches using actuarial techniques may not function well and we have to figure out a new approach to evaluate embedded option. Before introducing our method, we firstly make a review of recent researches into embedded option. Overall, existing studies on embedded options either focus on evaluation based on a generalized form or consider the pricing of a particular kind of contracts with their specific features.

Studies based on a generalized form were mainly about the evaluation method of embedded options. They all overlooked the specific features of different insurance contracts, for which these researches are less adaptable in practice. For researches that focus on particular insurance contracts, most of them are about unit-linked contracts and participating contracts. As for equity-linked products, large contributions have been made by financial and actuarial researchers since the pioneering work in [2,3]. They recognized the option-like features in equity-linked products and applied the option pricing theory on their evaluation model. Further researches in this direction were focusing on all kinds of options embedded in unit-linked contracts such as minimum guaranteed interest rate and payment (see [2,4-7]). The problem of valuing another kind of option embedded in equity-linked products, surrender option, has also been tackled, under different assumptions and by various methodologies (see [8-10]). However, all these researches left aside the effect of mortality, which obviously deviated from the reality.

As another kind of insurance contracts which also make up a significant part of the life insurance market, participating contracts also attracted great attention from researchers. Here is a brief review of researches into participating insurance. In the first place, scholars started their researches with single-period models. Briys and de Varenne focused on a particular participating insurance sold in French market and managed to valuate both liability and equity of this contract based on classical contingent claim theory (see [11]). In 1997, they made further effort on valuation of participating insurance with guaranteed interest rate and surrender option embedded. Based on this model, Grosen and Jørgensen adopted a barrier option framework to investigate the impact of regulatory intervention rules for reducing the insolvency risk of the policies (see [12]). However, all these researchers are not able to deal with "cliquet-style" contracts with annual bonus. Therefore, scholars also made attempt to construct multi-period models. Grosen and Jørgensen provided a multi-period model to valuate liability of participating insurance contract with "cliquet-style" bonus option and surrender option embedded (see [13]). After considering guaranteed interest rate, bonus option and surrender option, Grosen et al. proposed to use a finite difference approach for pricing contract (see [14,15]). Considering a specific participating scheme used in a particular country, Ballotta made a further step in this direction. They considered the valuation problem for one of the smoothing scheme commonly used by insurance companies in the UK and set up a market model based on the use of a Levy motion as relevant process for the value of underlying reference portfolio's returns (see [16]). Dealing with this same smoothing scheme, Ballota, Haberman and Wang further extended the techniques they developed earlier to allow for the default option. Researches mentioned above all focused on evaluation of participating policy with a given guaranteed interest rate. Aguilar and Xu provided a way to determine optimal guaranteed interest rates and premiums which maximize insurers' profit (see [17]). In his model, he assumed rates of return on assets were lognormally distributed and applied B-S formula to price surrender option. However, he did not allow for the possibility for insurers to default.

Aside from the embedded bonus options, some other embedded options have also drawn much attention. Herr and Kreer modeled a life insurance contract with surrender and paid-up options with underlying stochastic interest rates (see [18]). However, they did not take the effect of mortality into consideration. As for pricing contracts with surrender options, Albizzati and Geman have made some progress (see [19]). As a generalization and extension, intervention options which include all types of options that can be described by control by intervention were discussed in [20]. These options belong to the surrender and the free policy (paid-up policy) options. To deal with intervention options they assumed the simplest form of intervention, i.e., optimal stopping, which indicates exercise of all options was rational. Baione et al. analyzed both the term structure of interest and mortality rates role for evaluating a fair value of a life insurance business (see [21]). In their model, not only all the options mentioned above were allowed for, the option to annuities was also considered.

Meanwhile, some other scholars attempted to establish multi-decrement models. Bacinello considered the effect of different kinds of payment option, i.e., net single premium or regular premium (see [22,23]). Besides, mortality was also included in their model. Kling et al. provided a model to analyze the effect participating contract with "cliquet-style" guaranteed interest rate and default risk embedded had on liability, asset allocation and management decision of insurance company (see [24]). In addition, they pointed out that all these parameters were correlated.

In all these researches summarized above, they were all aiming at correctly valuing embedded options, based on which insurance companies could set aside adequate reserve to prevent insolvency risk. However, there are still some deficiencies of existing researches remaining to be discussed. First of all, contracts discussed in existing studies were limited to participating insurance and unit-linked insurance. Few comments have been made on another type of contracts which are also important, the universal products while in this article we are about to discuss this insurance products. Secondly, current researches rarely considered mortality risk in pricing embedded options. In fact, insurance products are designed to cover mortality risk. The effect of mortality risk on the value of embedded options could be too significant to neglect, for which we will take into account the effect of mortality in our model. Thirdly, existing studies have not allowed for different choices to make payments; for example, level premium or regular premium has rarely been discussed. In our research, we will take the assumption of level premium to construct our model. Fourthly, in existing studies, the effect of embedded options on solvency has not been analyzed quantitatively. Most of current researches just mentioned this effect qualitatively. For that reason, we will construct an insolvency model to analyze this effect. To sum it up, in order to deal with the four issues mentioned above, we will construct a valuation model for universal insurance with level premium, considering features of this insurance and the effect of mortality. Besides, we will also construct an insolvency model to figure out the relation between embedded option and insolvency risk.

The following is our modeling scheme. Focusing on universal life insurance, we first calculate level premium and construct the basic model of assets and liabilities. Here we made some changes on details to allow for mortality and the specific characters of universal insurance contracts. Then, we introduce interest rate guarantee and cancellable option into this model to make it more consistent with the reality of prevailing insurance policy. Based on the pricing of universal life insurance, the paper establishes the insolvency model. Finally, the authors use Monte Carlo simulation method to provide numerical results and sensitivity analysis to look into the relationship between price of contracts and insolvency risk, which could provide useful guidance in practice. The rest of the paper is structured as follows. In Section 2 we propose the valuation model and solvency model for universal contracts. In Section 3 we perform numerical simulation to verify our models. Finally, we make our conclusions in Section 4.

2. Valuation Model and Solvency Model of a Life Insurance Company with Universal Contracts.

2.1. Valuation model based on contingent claim theory and actuarial techniques. The basic framework is as follows. Insurers are assumed to operate in a continuous trading and frictionless financial market, so that there are no tax effects, no transaction costs, no short-sales constraints and all securities are divisible. We also ignore the effects of lapses in order to make solution tractable. These assumptions were also adopted in previous researches. The reasons are as follows. These assumptions are often used to describe the conditions in complete market, which is an ideal trading market. Conditions in complete market are much simpler than that in the real market. Therefore, we made these assumptions to simplify our model. In addition, results based on these assumptions could also be used to provide guidance in the real market because we believe that the difference between complete market and the real market do not make significant difference to our results.

Consider a universal life insurance endowment policy issued at time 0 and maturing T years after (at time T). Under this contract, the insurer[1] is obliged to pay a specified amount of money (benefit) to the beneficiary if the insured dies within the term of the contract or survives to the maturity date without surrender. In accordance with most insurance policy terms, we assume that, in case of death during the t-th year of contract, the benefit is paid at the end of the year, that is, at time t ($t = 1, \ldots, T$); otherwise, it is paid at maturity T. We assume that the contract is paid by level premiums[2], due at the beginning of each year, if the insured is alive. We denote by P the level premiums. To allow for the feature of universal life contracts, we denote by i the minimum guaranteed interest rate[3].

Model I: Valuation model for traditional endowment contracts

We first consider the simplest situation without effect of neither surrender nor participating policy, which we denote by model 1. We also denote by x the age of the insured at time 0 and by C_1 the specified benefit, payable in case of death. According to standard actuarial practice, the level premium, here we denote, is P_1 computed in the following way:

$$P_1 = C_1 P_{x:\overline{T}\backslash i} = C_1 \frac{A_{x:\overline{T}\backslash i}}{\ddot{a}_{x:\overline{T}\backslash i}} = C_1 \frac{\sum_{i=1}^{T-1} (1+i)^{-t} {}_{t-1|}q_x + (1+i)^{-T} {}_{T-1}p_x}{\sum_{i=1}^{T-1} (1+i)^{-t} {}_{t}p_x} \tag{1}$$

Here, i represents an (annually compounded) interest rate, ${}_{t-1|}q_x$ the probability that the insured is still alive at time t. As usual, these probabilities depend on the age of the insured and are extracted from the mortality table.

Under assumption of risk neutral, annual level premium can be obtained using risk free rate, denoted by r, instead of the actual interest rate. Equation (1) can be rewritten as follows:

$$P_1 = C_1 P_{x:\overline{T}\backslash r} = C_1 \frac{A_{x:\overline{T}\backslash r}}{\ddot{a}_{x:\overline{T}\backslash r}} \tag{2}$$

Here, $C_1 A_{x:\overline{T}\backslash r}$, denoted by U^{P_1} for convenience, represents the net single premium that is equivalent with the annual level premium. We can interpret this net single premium as the value of bond covering mortality risk.

[1]For simplicity, we assume the insured is also the applicant.
[2]For simplicity, we do not allow for varying premium.
[3]Minimum guaranteed rate guarantees interest of policyholders, embodying the security of insurance.

Model 2: Valuation model for universal insurance contracts with guaranteed interest rate

Observe that the premium in model 1 makes the expected value at time 0 of the benefit C_1 of a standard life insurance endowment policy, discounted from the random time of payment, equal to the expected value at time 0 of the stream of constant annual premiums P_0, discounted as well. Then, on the ground of the first-order bases, P_0 makes the contract "fair" at inception. However, the guaranteed interest rate specified in universal contracts forces the benefit to vary, year by year, since a bonus is annually granted to the policy and used to purchase additional insurance. To allow for this, we construct another model, called model 2 for convenience, based on model 1.

Prior to the introduction of model 2, we have to give definitions of some notions. We first denote by P_2 the constant annual premium payable at the beginning of each year, provided that the insured is alive. Note that as benefits are no longer constant, this premium will be different from the premium in model 1, P_1. Similarly, we denote by C_t, $t = 2, 3, \ldots, T$ the benefit payable at time t, given by the initial C_1 plus the additional benefits acquired from time 1 to time $t-1$. Assuming that insurers are risk neutral and mortality risks and financial risks are independent, we could employ a two-stage scheme firstly to calculate the net single premium and then to obtain annual premium. The expected value at time 0 of the benefit C_t, discounted from random time of payment, can be expressed as follows:

$$\pi(C_t) = E^Q[C_t(1+r)^{-t}] \tag{3}$$

Based on this, the net single premium is easily obtained.

$$U^{P_2} = \sum_{t=1}^{T-1} \pi(C_t)\,{}_{t-1|}q_x + \pi(C_T)\,{}_{T-1}p_x \tag{4}$$

Note that the difference between the net single premiums in model 1 and model 2 actually represents the required compensation for insurance company to cover the cost of offering a minimum guaranteed interest rate.

The annual premium in model 2 can be obtained in the following way:

$$P_2 = U^{P_2} \Big/ \ddot{a}_{x:\overline{T}\backslash r} \tag{5}$$

The next problem to obtain an analytical expression for annual premium P_2 is to figure out how all these changes in benefit are settled and thereby determine the expression for C_t.

(1) Modeling varying benefit

Here we make the assumption that benefit equals the initial benefit C_1 plus the value of policyholder's account, or equivalently equals the liability reserve. This is also consistent with the terms in universal insurance contracts. To see how benefit changes is actually to see how value of liability reserve changes. According to standard actuarial practice, liability reserve at time t, denoted by ${}_tV$, $t = 1, 2, \ldots, T-1$, can be computed in the following way:

$$\begin{aligned} {}_tV &= C_t A_{x+t:\overline{T-t}\backslash i} - P_2 \ddot{a}_{x+t:\overline{T-t}\backslash i} \\ &= C_t \left[\sum_{h=1}^{T-t} (1+i)^{-h}\,{}_{h-1|}q_{x+t} + (1+i)^{-T}\,{}_{T-t}p_{x+t}\right] - P_2 \sum_{h=1}^{T-t-1} (1+i)^{-h}\,{}_hp_{x+t} \end{aligned} \tag{6}$$

where ${}_{h-1|}q_{x+t}$ represents the probability that the insured dies within the $(t+h)$-th year of contract, conditioned on the event that he or she is alive at time t, and ${}_hp_{x+t}$ is the probability that the insured is still alive at time $t+h$, conditioned on the same event.

Note that relation (6) would not provide a proper way to compute the policy reserve, because it is established as if premiums and benefits remained constant after time t. However, this relation is actually used in practice as a starting point for defining the accumulation mechanism. Here we use P_1 in place of P_2 to compute ${}_tV$ in order to split the effect of guaranteed interest rate on insurance coverage. As mentioned above, benefit equals the initial benefit C_1 plus the value of policyholder's account. Under usual assumption, the former is constant. As for the value of policyholder's account, premiums collected, less regular expenses and risk cost, are credited into the policyholder's account and reinvested in financial market. We denote by $A(t)$ the portfolio invested and g_t the rate of return on $A(t)$. We further assume that all return on $A(t)$ should be included in policyholder's account. Besides, a minimum guaranteed interest rate, denoted by r_g, is provided in a universal insurance product, which ensures value of policyholder's account increase at a rate of $\max\{r_g, g_t\}$. It can be demonstrated that the varying benefit is expressed in the following way:

$$\begin{aligned} C_t = {} & C_1 + \left(P_1 - A^1_{x:\overline{1}|}\right) \max\{1+r_g, 1+g_1\} \cdots \max\{1+r_g, 1+g_t\} \\ & + \left(P_1 - A^1_{x+1:\overline{1}|}\right) \max\{1+r_g, 1+g_2\} \cdots \max\{1+r_g, 1+g_t\} \\ & + \left(P_1 - A^1_{x+2:\overline{1}|}\right) \max\{1+r_g, 1+g_3\} \cdots \max\{1+r_g, 1+g_t\} \\ & + \cdots + \left(P_1 - A^1_{x+t-1:\overline{1}|}\right) \max\{1+r_g, 1+g_t\}, \;\; t = 1, 2, 3, \ldots, T \end{aligned} \tag{7}$$

The equation above actually converts the effect of guaranteed interest rate to a constrained varying insured amount, which could also be used to compute the policy reserve.

(2) The asset side of the balance sheet

Note that either policy reserve or varying benefit depends strongly on the value of the invested portfolio, i.e., the asset of insurance company. To price contracts with varying benefit we need to know the behavior of asset price.

Recall the description of the structure of the contract, the annual premium P is collected at the beginning of each year and will be invested during the following year together with the policy reserve accumulated till the end of last year. The insurance company is assumed to invest the money in a well-diversified asset portfolio. We make no assumptions regarding the composition of this portfolio with respect to equities, bonds, real estate. Instead, we simply assume the asset portfolio to follow the traditional geometric Brownian motion, so as to make our model easier to deal with

$$\begin{aligned} & dA(t_s) = rA(t_s)dt + \sigma A(t_s)dW(t_s) \\ & A(t_0) = A(t-1) + P_1 - A^1_{x+t-1:\overline{1}|}, \;\; t \in [t-1, t), \;\; t = 1, 2, \ldots, T, \;\; A(0) = 0 \end{aligned} \tag{8}$$

under the risk neutral probability Q. The parameter $r \in R^+$ is the risk-free rate of interest and $\sigma \in R^+$ is the portfolio volatility. $W(t)$ is a standard Brownian motion under Q. According to [25], this stochastic differential equation has a solution given by

$$A(t) = \left[A(t-1) + P - A^1_{x+t-1:\overline{1}|}\right] \exp\left\{\left(r - \frac{1}{2}\sigma^2\right) + \sigma W^Q(t_s)\right\} \tag{9}$$

And the annual rate of return, g_t, could be expressed as follows:

$$g_t = \left(r - \frac{1}{2}\sigma^2\right) + \sigma W^Q(t_s) - 1 \tag{10}$$

Using a simulation technique, we could obtain the value at time T of asset, $A(T)$.

Model 3: Valuation model for universal contracts with guarantees and surrender option

All discussions above left aside the effect of surrender. Here we introduce this important factor into our valuation model. For convenience, the model considering surrender is called model 3 in this paper. Surrender option provides policyholder the right to terminate the contract ahead of time and acquire an amount of money called surrender value, usually less than the overall premium that he or she has paid. By its definition, we could reason that surrender option is American-style option since policyholder could exercise the option anytime when he or she thinks the expected value of a contract is less than the surrender value.

We firstly introduce our assumptions on surrender. In order to comply with the surrender practice, we assume that surrender could only be processed at the end of each year, i.e., at time $t = 1, 2, \ldots, T$. As required by most insurance regulations, surrender value a policyholder can get equals to his or her account value minus some amount of money called the surrender charge. In this paper, surrender value is assumed to equal the value of policyholder's account, i.e., ${}_tV - C_1$, which could be computed using Equation (7). The expression for ${}_tV$ is as follows:

$$\begin{aligned} V_t = {} & \left(P_1 - A^1_{x:\overline{1}|}\right) \max\{1+r_g, 1+g_1\} \cdots \max\{1+r_g, 1+g_t\} \\ & + \left(P_1 - A^1_{x+1:\overline{1}|}\right) \max\{1+r_g, 1+g_2\} \cdots \max\{1+r_g, 1+g_t\} \\ & + \left(P_1 - A^1_{x+2:\overline{1}|}\right) \max\{1+r_g, 1+g_3\} \cdots \max\{1+r_g, 1+g_t\} \\ & + \cdots + \left(P_1 - A^1_{x+t-1:\overline{1}|}\right) \max\{1+r_g, 1+g_t\}, \quad t = 1, 2, 3, \ldots, T \end{aligned} \tag{11}$$

Note that the choice to surrender or not at time τ is based on all information available until time τ and the optimal choice must yields the highest expected value at time 0. Based on that, expected benefit or surrender value at time 0 discounted from a random time τ could be computed as follows:

$$\tau U^{P_3} = \sum\nolimits_{t=1}^{\tau} \pi(C_t)_{t-1|}q_x + \pi({}_\tau V)_\tau p_x \tag{12}$$

We could further obtain the net single premium,

$$U^{P_3} = E^Q\left[\sup\nolimits_{\tau\in\Gamma[0,T-1]}({}_\tau U^{P_3})\right] \tag{13}$$

Here τ is an optimal stopping time with domain on $[0, T-1]$. Solving this problem involves pricing compound option, yielding no analytical solution. We will employ a Least Square Monte Carlo Simulation technique (see [26]) to find the answer. It can be easily seen that the value of surrender option equals the difference between net single premiums in model 2 and model 3.

2.2. **Solvency model construction.** We have already constructed the valuation model for universal insurance contracts, based on which we could further consider forming a solvency analysis model. According to "Administrative Provisions on the Solvency of Insurance Companies", actual capital, denoted $AC(t)$, is the difference between recognized asset and recognized liability. Here we make the simple assumption that all assets and liability of insurance companies are admissible. Then the actual capital could be computed as follows:

$$AC(t) = A(t) - {}_tV \tag{14}$$

According to "Administrative Provisions on the Solvency of Insurance Companies", the minimum capital insurers must possess equaling 4% of the value of policyholder's account $P(t)$. Thus, the solvency margin ratio, which is defined to be the ratio of actual capital to the minimum capital, could be computed in the following way:

$$SMR(t) = AC(t)/MC(t) \tag{15}$$

3. Numerical Results.

3.1. **Simulation based on valuation model.** It is now time to find a solution to the valuation of the specific liability according to the model we have constructed. Form a computational point of view, numerical valuation via Monte Carlo simulation is a possibility referring to path-dependent feature of ${}_tV$. To price universal insurance products, we firstly repeatedly simulate the asset value at time t, i.e., $A(t)$, and its corresponding rate of return, g_t. Then we could obtain the varying benefit and policy reserve at time t, C_t and ${}_tV$ respectively. Applying the contingent claim theory, $\pi(C_t)$ and $\pi({}_tV)$ could also be determined. With sufficiently enough number of simulations, the average offers an asymptotic unbiased estimator of the real value.

The premium determined in model 1 is used in simulation of asset price. To further determine the premiums in other models, we adopt the following procedure. Firstly, we calculate the premium in model 1, P_1, and then solve model 2 for U^{P_2} as if P_2 was determined and equals to P_1, and use the relationship between U^{P_2} and to determined the real P_2. The premium in other models could be obtained in the same way. Based on these results, we could compute the value of guarantee interest rate and surrender option.

Before exhibiting our results, we need to make specification of the data and parameters used in our simulation. Firstly, data of mortality risk are extracted from "CHINA LIFE INSURANCE MORTALITY TABLE(2000-2003)"-CL1. We also assume a series of possible numbers from 0.8 to 1.2 as the mortality modifying factors to analyze the effect of mortality. Moreover, to compare our model with the one which does not allow for mortality, we include 0 as an extreme scenario in our simulation. As for the contract parameters, they are not interested parameters in our research so we just adopt the assumptions used in previous researches. More specifically, the initial insured amount is assumed to be 10000 and the term of contract is 10 years. Two other contract parameters (x, r_g) are interested variable in this paper and are assumed to take on a series of reasonable numbers in a later part of this section. Since most purchasers of this universal insurance are mid age people who earn regular income and fear for unexpected losses affecting their families, we assume that the age, x, are in the range between 40 and 60. The baseline age in our simulation is 40. Due to the guaranteed interest rate is no greater than the limited 2.5% according to legislation rules, we specify r_g to be 2.5% in our simulations. Aside from contract parameters, there are two other important market parameters (σ, r) which we have to consider carefully in accordance with the market situation in China. The investment of insurance company is required to be safe enough to prevent the loss from insolvency, which means that asset investment risk should be low, regulated by China Insurance Regulatory Commission. Thus, we define the volatility parameter σ to be 15% in this study.

Here we repeat the baseline parameters for convenience. Based on these parameters and 100000 simulations, we could estimate premiums in all three models.

$$U^{P_1} = 6776, \quad U^{P_2} = 10148, \quad {}_\tau U^{P_3} = 16420, \quad P_1 = 808.5, \quad P_2 = 1209.1, \quad P_3 = 1679.5$$

Furthermore, we can obtain the value of guaranteed interest rate and surrender options, $U^{P_2} - U^{P_1} = 3372$ and $U^{P_3} - U^{P_2} = 6272$, respectively. It can be seen that the value of guarantees are quite high, taking a 49.8% proportion of U^{P_1}. In addition, the value of surrender option takes the most part of the whole net single premium. For that reason, measures should be taken to reduce the rate of surrender.

Now we focus on parameters we are interest in, age, guaranteed interest rate, risk free rate and volatility. In a second part of our simulation, these parameters are assumed to take on a series of reasonable numbers. Numerical results based on these varying parameters are contained in Table 1, which could be divided into four parts, used in sensitivity analysis of these four variables respectively.

In the interpretation of Table 1, we first discuss the effect of guaranteed interest rate and surrender option. Consist with the results we obtained in the situation with baseline parameters, the effect of surrender option and interest guarantees are too significant to neglect. Seen from the first part of Table 1, with increase in the age of insured, premium to purchase classical endowment contract, P_1, increases. So does the premium for model 2, P_2. However, increase in age leads to a decrease in value of surrender option while a overall increase in P_3. Based on this information, it is suggested that as ages increase insured tends more to consider insurance contract as an investment and less to surrender.

The second part of Table 1 suggests that increase in risk free rate results in decreases of premiums in model 1 and model 3. This can be explained by the fact that as risk free rate decreases value of guarantees decreases and value of surrender option increases.

From the third part of Table 1, we can reason that increase in guaranteed interest rate leads to a slightly increase in value of guarantee. It is implied that with a high risk free rate the effect of guaranteed rate on the value of guarantee is insignificant. However,

Table 1. Option values varying with parameters

	value	U^{P_1}	U^{P_2}	U^{P_3}	Guaranteed interest rate	Surrender option
x	40	808.5	1210.7	1969.8	402.2	759.2
	45	811.8	1222.0	1973.6	410.2	751.5
	50	817.1	1240.1	1976.4	423.0	736.3
	55	827.3	1276.5	1980.0	449.2	703.4
	60	848.4	1346.4	1994.2	497.9	647.8
r	2%	902.7	1115.6	2267.5	212.9	1151.9
	4%	808.5	1210.7	1971.4	402.2	760.7
	6%	723.6	1311.2	1718.1	587.5	406.9
	8%	647.3	1416.9	1504.5	769.6	87.6
	10%	578.8	1329.6	1329.6	750.8	0.0
	15%	437.1	979.8	979.8	542.7	0.0
	20%	330.2	738.9	738.9	408.7	0.0
r_g	1%	808.5	1209.7	1926.3	401.2	716.6
	2%	808.5	1210.1	1954.7	401.6	744.7
	3%	808.5	1210.8	1986.0	402.3	775.2
	4%	808.5	1211.4	2019.4	402.9	808.0
	5%	808.5	1211.4	2055.2	402.9	843.8
σ	10%	808.5	1210.1	1850.3	401.6	640.2
	20%	808.5	1210.2	2106.9	401.8	896.7
	30%	808.5	1213.6	2433.8	405.1	1220.2
	40%	808.5	1216.0	2843.6	407.5	1627.7
	50%	808.5	1215.6	3350.5	407.1	2134.9
Mortality Modifying Factor	0	800.9	1184.8	1963.4	384.0	778.6
	0.8	807.0	1205.2	1968.9	398.2	763.7
	0.9	807.7	1207.7	1969.6	400.0	761.9
	1	808.5	1210.3	1970.3	401.8	760.0
	1.1	809.3	1212.8	1971.0	403.6	758.1
	1.2	810.0	1215.4	1971.7	405.4	756.3

as guaranteed interest rate increases, the value of surrender option tends to increase significantly due to the increasing surrender rate.

From the forth part of Table 1, it can be seen that higher volatility does not necessarily bring a significantly higher value of guarantees. Nevertheless, the value of surrender option increases dramatically, which, again, lays emphasize on the effect of surrender on universal products. Without sufficient consideration of this effect, the determined premium could be way too deficient to guarantee the insurer's solvency.

From the last part of Table 1, we also learn that the effect of death rate is significant. The higher the mortality rate, the higher the premium and the higher the guaranteed interest rate option worth. The reason lies in the fact that higher death rate always indicates higher chance to obtain the benefit earlier, which results in a higher present value of this contract. While in the contrary, higher mortality rate may go with lower value in surrender option. This may be explained by the higher present value of this contract caused by higher death rate, which reduces the incentive for policyholders to surrender.

3.2. **Simulation based on solvency analysis model.** We are about to analyze the effect of some extraneous variables on solvency. Interested variables include guaranteed interest rate, age, market parameters. Based on 100000 simulations each, we estimate the actual capital, minimum capital and SMR in each scenario and analyze the effect of these interested parameters on solvency.

(1) Effect of age on solvency

From Figure 1, it can be seen that as the insured grows old there is a slight increase in both actual capital and solvency marginal ratio, which implies an insignificant effect of age on solvency. Besides, when the expiring date draws near, actual capital increases and SMR decreases drastically. This can be explained by the fact that when contracts are about to expire insurer must prepare less reserves. Consequently, the actual capital increases significantly compared with the slight decrease in minimum capital, thus leading to a rapidly decrease in SMR. To deal with this increasing solvency risk, more policy reserve is required close to expiring date.

(2) Effect of guaranteed interest rate on solvency

To better convey the effect of guaranteed interest rate on solvency, we plot the results in Figure 2. Seen from Figure 2, the effect of minimum guaranteed interest rate is significant and increases as time goes by. It is also suggested that minimum guaranteed interest rate affect solvency marginal ratio in a different way. Firstly, SMR in each situation decreases

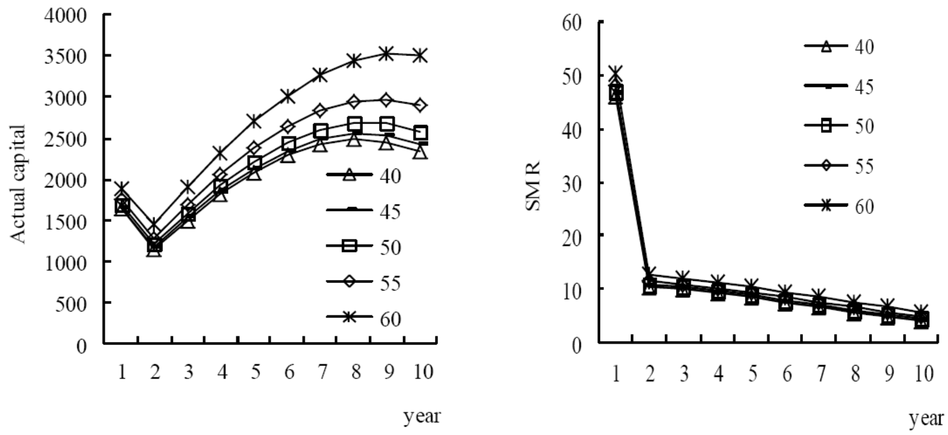

Figure 1. Effect of age on solvency

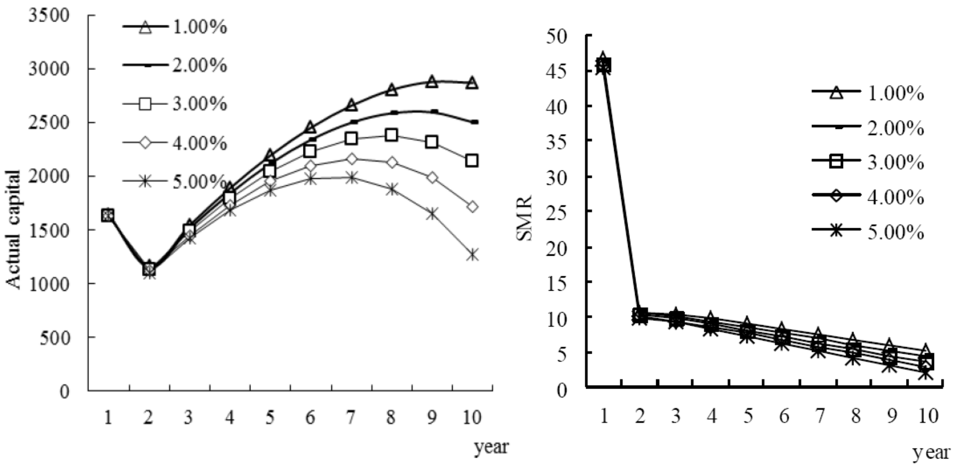

FIGURE 2. Effect of guaranteed interest rate on solvency

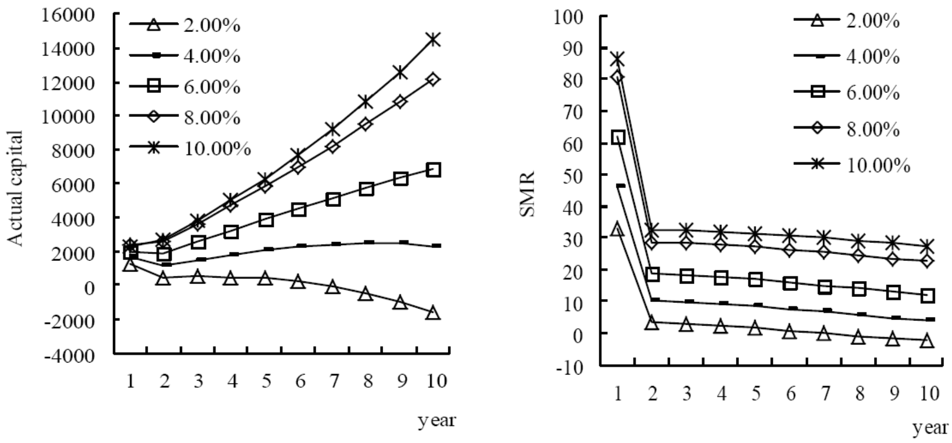

FIGURE 3. Effect of risk free rate on solvency

over time. Secondly, in each situation the speed at which SMR decreases differs a little bit. The lower the guaranteed interest rate, the slower SMR decreases. Hence, it can be reasoned out that guaranteed interest rate does affect solvency and the degree of this influence varies from one policy anniversary to another. Also note that the results presented here differ from those obtained assuming no effect of mortality, which means the latter are actually inaccurate.

(3) Effect of risk free rate on solvency

The results for analyzing effect of risk free rate on solvency are plotted in Figure 3. Note that there exist a great effect of risk free rate on both actual capital and SMR, especially in case when contracts are about to expire. This is a similar situation with the one we were confronted with when dealing with guaranteed interest rate, only the effect is greater. With a low risk free rate actual capital decreases to a negative level while with a high one actual capital tends to increase rapidly. To reason this, just consider the consequences of increasing risk free rate. Increase in risk free rate leads to increase in value of options embedded and thus the value of asset. Eventually, this increase will result in an increase in solvency of insurance company and necessarily an increase in SMR. However, if risk free rate is not sufficiently high to keep the asset value higher than liability, this gap would get widening, indicating that a capital injection is necessary.

(4) Effect of volatility on solvency

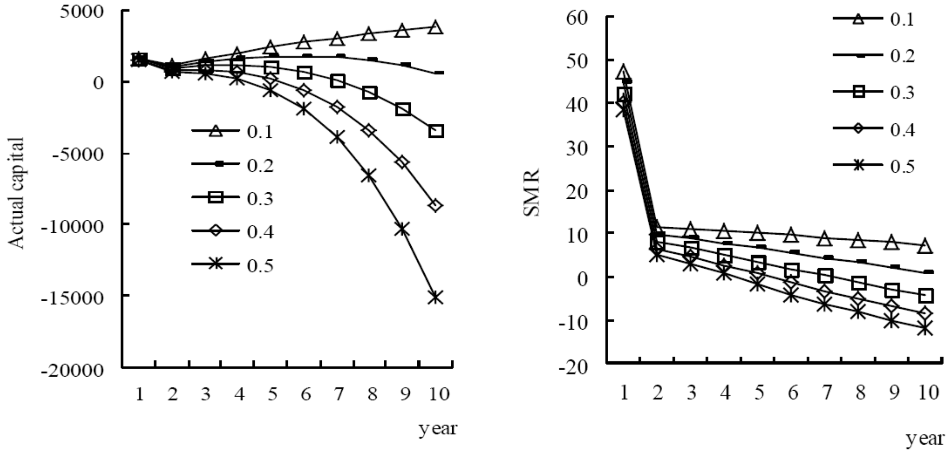

Figure 4. Effect of volatility on solvency

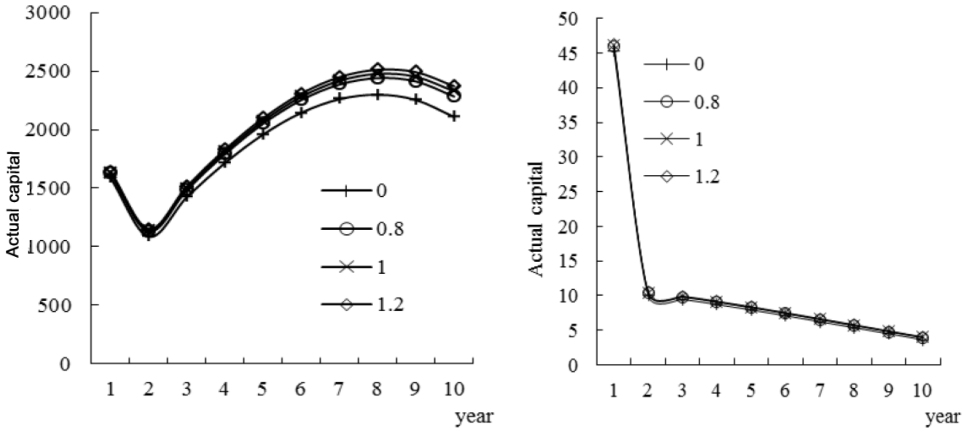

Figure 5. Effect of mortality risk on solvency

The forth part of this section is a discussion on the effect of volatility on solvency. From Figure 4, we can learn that volatility affects solvency in a similar but more notable way. With a high volatility, the actual capital and SMR get very negative, indicating a high risk and an urgent requirement of capital injection.

(5) Effect of mortality rate

As mentioned above, we looked the mortality risk as a key variable in constructing our models. Now let us take a look at the effect of mortality on solvency, which is presented in Figure 4. As it can be seen from Figure 3, the higher the mortality rate, the higher the insolvency risk. This is consistent with our knowledge since higher death rate indicates higher insurance liabilities, which necessarily implies higher insolvency risk.

4. **Conclusions.** In the case of a dramatically changing environment, the value of embedded options gets into the state of “in the money” from the state of “out of the money”, thus increasing the insurer’s liability. Among all embedded options, guaranteed interest rate and surrender are two most important factors when determining the value of universal contracts. Ever since the international Accounting Standard Board (IASB) led a project aiming at implementing new accounting standards for insurance, fair valuation of insurance contract has drawn great attention. With the limitation of existing studies, it requires further push in this direction, especially on the problem of pricing insurance contracts on a fair value basis.

Considering effects of mortality, surrender, guarantees, in this paper we introduced contingent claim theory into the classical actuarial framework and construct a fair valuation model for universal insurance contract. Starting from a classical endowment, we adopt a three step approach to model the payoff and compute the premium of universal contract. Based on the pricing of universal life insurance, the paper establishes the insolvency model. Finally, Monte Carlo simulations are conducted to provide numerical results and sensitivity analysis. In order to figure out the effect of mortality rate on embedded option and insolvency risk, we also performed a mortality scenario test by introducing a parameter called "mortality modifying factor", from which we could see the difference between our research and other researches where mortality effect is not considered.

The main numerical results are summarized as follows. In the first place, as a part of the whole value of universal contract, the value of embedded options accounts for a high proportion. Therefore, the effect of guarantees and surrender should be fully considered to avoid under-pricing and insolvency. Otherwise in the case of a dramatically changing environment, the insurer could suffer from enormous losses and even go bankrupt. In addition, value of embedded options is affected by extraneous variables like age, risk free rate, guaranteed return and volatility. As any of these variables increase, the option of guaranteed interest rate tends to appreciate. Not only increase in guaranteed return or volatility but also decrease in age or risk free rate will lead to an increase in the value of surrender option. Results also indicate that mortality rate has great influence on embedded options. As mortality rate increases, the value of option of guaranteed interest rate increases while the value of surrender option decreases. This could be explained by the fact that higher death rate always means earlier compensation from holding this insurance contract, which might increases the expected value of the guaranteed interest rate and also the value of universal insurance contract. The increase in universal insurance contracts further reduces the incentive to surrender, thus decreasing the value of surrender option.

Secondly, numerical research also shows that both actual capital and solvency margin ration are deeply affected by extraneous variables like age, risk free rate, guaranteed return and volatility. Analysis of these effects could offer a useful guidance in risk management. Increase in age may lead to an increase in solvency ratio. It is because insurance companies facing older group of insurant will set aside more reserve than those facing younger one. Besides, increase in risk free rate always indicates an improvement of investment environment, which consequently contributes to a better solvency status. On the contrary, as guaranteed return and volatility respectively represent the liability and risk that insurance companies are facing, the increase in any of them will cause a decrease in solvency ratio.

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REFERENCES

[1] A. M. Santomero and D. F. Babbel, Financial risk management by insurers: An analysis of the process, *Journal of Risk and Insurance*, pp.231-270, 1997.

[2] M. J. Brennan and E. S. Schwartz, The pricing of equity-linked life insurance policies with an asset value guarantee, *Journal of Financial Economics*, vol.3, no.3, pp.195-213, 1976.

[3] P. P. Boyle and E. S. Schwartz, Equilibrium prices of guarantees under equity-linked contracts, *Journal of Risk and Insurance*, vol.44, no.4, pp.639-660, 1977.

[4] A. R. Bacinello and F. Ortu, Pricing equity-linked life insurance with endogenous minimum guarantees, *Insurance: Mathematics and Economics*, vol.12, no.3, pp.245-257, 1993.

[5] J. A. Nielson and K. Sandmann, Equity-linked life insurance – A model with stochastic interest rates, *Discussion Paper Serie B*, University of Bonn, Germany, no.291, 1995.

[6] T. Moller, Hedging equity-linked life insurance contracts, *North American Actuarial Journal*, vol.5, no.2, pp.79-95, 2001.

[7] H. Xu, *Valuation Problem for an Equity-Linked Policy with Guaranteed Interest Rate*, Fudan University, Shanghai, 2001.

[8] A. Grosen and P. L. Jørgensen, Valuation of early exercisable interest rate guarantees, *Journal of Risk and Insurance*, vol.64, no.3, pp.481-503, 1997.

[9] J. Chen, Asymptotic behavior of free boundary in the model of the value of equity-linked policy, *Journal of Fudan University*, vol.43, no.3, 2004.

[10] W. Shen and H. Xu, The valuation of unit-linked policies with or without surrender options, *Insurance: Mathematics and Economics*, vol.36, no.1, pp.79-92, 2005.

[11] E. Briys and F. de Varenne, Life insurance in a contingent claim framework: pricing and regulatory implications, *The GENEVA Papers on Risk and Insurance-Theory*, vol.19, no.1, pp.53-72, 1994.

[12] A. Grosen and P. L. Jørgensen, Life insurance liabilities at market value: An analysis of insolvency risk, bonus policy, and regulatory intervention rules in a barrier option framework, *Journal of Risk and Insurance*, vol.69, no.1, pp.63-91, 2002.

[13] A. Grosen and P. L. Jørgensen, Fair valuation of life insurance liabilities: The impact of interest rate guarantees, surrender options, and bonus policies, *Insurance: Mathematics and Economics*, vol.26, no.1, pp.37-57, 2000.

[14] B. Jensen, P. L. Jørgensen and A. Grosen, A finite difference approach to the valuation of path dependent life insurance liabilities, *The GENEVA Papers on Risk and Insurance-Theory*, vol.26, no.1, pp.57-84, 2001.

[15] A. J. Tanskanen and J. Lukkarinen, Fair valuation of path-dependent participating life insurance contracts, *Insurance: Mathematics and Economics*, vol.33, no.3, pp.595-609, 2003.

[16] L. Ballotta, A Lévy process-based framework for the fair valuation of participating life insurance contracts, *Insurance: Mathematics and Economics*, vol.37, no.2, pp.173-196, 2005.

[17] P. R. C. Aguilar and C. Xu, Design life insurance participating policies with annual premiums using optimization techniques, *Proc. of the 4th International Conference on Innovative Computing, Information and Control*, Kaohsiung, pp.1140-1143, 2009.

[18] H. O. Herr and M. Kreer, Zur bewertung von optionen und garantien bei lebensversicherungen, *Blätter der DGVFM*, vol.24, no.2, pp.179-193, 1999.

[19] M. O. Albizzati and H. Geman, Interest rate risk management and valuation of the surrender option in life insurance policies, *Journal of Risk and Insurance*, vol.61, no.4, pp.616-637, 1994.

[20] M. Steffensen, Intervention options in life insurance, *Insurance: Mathematics and Economics*, vol.31, no.1, pp.71-85, 2002.

[21] F. Baione, P. De Angelis and A. Fortunati, On a fair value model for participating life insurance policies, *Investment Management and Financial Innovations*, vol.3, no.2, pp.105-115, 2006.

[22] A. R. Bacinello, Fair valuation of a guaranteed life insurance participating contract embedding a surrender option, *Journal of Risk and Insurance*, vol.70, no.3, pp.461-487, 2003.

[23] A. R. Bacinello, Pricing guaranteed life insurance participating policies with annual premiums and surrender option, *North American Actuarial Journal*, vol.7, no.3, pp.1-17, 2003.

[24] A. Kling, A. Richter and J. Ruβ, The interaction of guarantees, surplus distribution, and asset allocation in with-profit life insurance policies, *Insurance: Mathematics and Economics*, vol.40, no.1, pp.164-178, 2007.

[25] J. M. Harrison and D. M. Kreps, Martingales and arbitrage in multiperiod securities markets, *Journal of Economic Theory*, vol.20, no.3, pp.381-408, 1979.

[26] F. A. Longstaff and E. S. Schwartz, Valuing American options by simulation: A simple least-squares approach, *Review of Financial Studies*, vol.14, no.1, pp.113-147, 2001.

[27] L. Ballotta, S. Haberman and N. Wang, Guarantees in with-profit and unitized with-profit life insurance contracts: Fair valuation problem in presence of the default option, *Journal of Risk and Insurance*, vol.73, no.1, pp.97-121, 2006.

[28] N. Gatzert and H. Schmeiser, Assessing the risk potential of premium payment options in participating life insurance contracts, *Journal of Risk and Insurance*, vol.75, no.3, pp.691-712, 2008.