

SIMULATING FUZZY NUMBERS FOR SOLVING FUZZY EQUATIONS WITH CONSTRAINTS USING GENETIC ALGORITHMS

FENG-TSE LIN

Department of Applied Mathematics
Chinese Culture University
Yangminshan, Taipei 111, Taiwan
ftlin@faculty.pccu.edu.tw

Received October 2008; revised March 2009

ABSTRACT. *This study investigates applying genetic algorithms (GAs) to solve fuzzy equations without defining membership functions for fuzzy numbers, neither using the extension principle and interval arithmetic and α -cut operations, nor using a penalty method for constraint violations. Three experimental examples were employed to illustrate the effectiveness of the proposed GA approach in solving fuzzy equations with constraints. An essential issue of applying GAs for obtaining better solution to the problem is the parameter settings including the probability of crossover, the probability of mutation, and the number of generations. Experimental results show that the proposed GA approach obtains very good solutions within the given bounds of each uncertain variable in the problems. The fuzzy concept of the GA approach is different, but provides better solutions than classical fuzzy methods.*

Keywords: Fuzzy equations, Fuzzy shortest path, Fuzzy knapsack, Fuzzy linear programming, Genetic algorithms

1. Introduction. A fundamental problem in the area of fuzzy set theory is solving fuzzy equations for optimization problems [3]. This is because the exact values of the coefficients of optimization problems in many practical applications are either vague or ambiguous due to the subjective viewpoint of a decision-maker (DM). To deal with this kind of uncertain variables, fuzzy sets provide a powerful tool to model the problem. The coefficients of the problem are thus replaced by fuzzy numbers that produce fuzzy equations to be solved [2]. Each fuzzy set is uniquely defined by one particular membership function for each uncertain variable. The membership function essentially embodies all fuzziness for a particular fuzzy set; its description is the essence of a fuzzy property or fuzzy operation.

The motivation of this study is described as follows. Consider a quadratic algebraic expression $ax^2 + bx$ for a, b real parameters and x a real variable. Substituting fuzzy numbers \tilde{A} , \tilde{B} , and \tilde{X} into $ax^2 + bx$ for a, b , and x , respectively, yields the fuzzy expression $\tilde{A}\tilde{X}^2 + \tilde{B}\tilde{X}$. Two classical fuzzy methods for evaluating fuzzy equations in the literature are summarized below [2,3]. The first method involves using the extension principle for finding the value of $\tilde{A}\tilde{X}^2 + \tilde{B}\tilde{X}$. The second method is using α -cuts and interval arithmetic for evaluating $\tilde{A}\tilde{X}^2 + \tilde{B}\tilde{X}$. Given any two fuzzy numbers \tilde{A} and \tilde{B} , the following operations hold:

$$(1) (\tilde{A}\tilde{B})(\alpha) = \tilde{A}(\alpha)\tilde{B}(\alpha), \text{ and}$$

$$(2) (\tilde{A} \pm \tilde{B})(\alpha) = \tilde{A}(\alpha) \pm \tilde{B}(\alpha)$$

$\tilde{A}\tilde{B}$ and $\tilde{A} \pm \tilde{B}$ are computed using the extension principle, and $\tilde{A}(\alpha)\tilde{B}(\alpha)$, $\tilde{A}(\alpha) \pm \tilde{B}(\alpha)$ are found using interval arithmetic. Let $\tilde{Y}(\alpha) = \tilde{A}(\alpha)\tilde{X}(\alpha)\tilde{X}(\alpha) + \tilde{B}(\alpha)\tilde{X}(\alpha)$, $0 \leq \alpha \leq 1$. Evaluating fuzzy algebraic expression using interval arithmetic and α -cuts can produce