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THE MODELING AND SIMULATION OF A CLASS OF HYPERCHAOS

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Abstract. *A class of hyperchaos is modeled based on a modified smooth three-dimensional chaotic system by adding a feedback controller. The new hyperchaotic system has two very large positive Lyapunov exponents in a quite wide parameter range through observing the Lyapunov exponents spectra, which indicates that it has very complex nonlinear dynamics. Some simulations of the phase portraits of hyperchaotic attractors are made to show the complicated system orbits. The Poincaré section analysis shows that the new system bifurcates and folds simultaneously in several various directions. The frequency spectra are also plotted to show that the system has an extremely broad bandwidth when it is hyperchaotic, which implies that the system has high complexity and randomness, and may have great potential applications in chaos-needed fields such as secure communication and encryption.*
Keywords: Hyperchaos, Three-dimensional chaotic system, Lyapunov exponent, Poincaré section, Frequency spectra

1. **Introduction.** The hyperchaotic system has at least two positive Lyapunov exponents, indicating that its dynamics are expanded in more than one direction simultaneously. For the autonomous continuous system, the dimension of a hyperchaotic attractor must be at least four, however, for a chaotic attractor, three-dimension is enough and it has just a single positive Lyapunov exponent. Therefore, compared with ordinary chaotic system, hyperchaotic system has more complicated and richer dynamics so as to be better used in many chaos-needed fields. For example, in the chaotic secure communication, a chaotic signal is utilized to mask the transmitted messages, and one had believed that messages under this situation were highly safe until 1995, when Perez and Cerdeira proved that the chaotic signal can be easily extracted sometime [1]. That is because chaotic signal has just one positive Lyapunov exponent, which means its dynamics are expanded in one direction and its trajectory is not very disordered. Fortunately, higher dimensional hyperchaotic system can overcome this problem because of its increasing randomness and higher unpredictability [2].

Hyperchaos seems to be more valuable and promising. Recently, the generation of hyperchaos and the hyperchaotic circuit realization have attracted researchers' increasing attention. For hyperchaos, two aspects should be mentioned as follows

(i) The minimal dimension of the phase space that embeds the hyperchaotic attractor should be at least four, which requires the minimum number of coupled first-order autonomous ordinary differential equations to be four.
(ii) The number of terms in the coupled equations that give rise to instability should be at least two, in which one should be a non-linear function [3].