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AN EXTENSION OF GENERALIZED BILINEAR TRANSFORMATION FOR DIGITAL REDESIGN

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Abstract. *In this paper, the existing digital redesign method, Generalized Bilinear Transformation (GBT), is extended so that large overshoot is suppressed while comparatively large sampling period is obtained in the digital control system. The extension is established by introducing piecewise or multiple discretization parameters in GBT. Several simulation examples are given to show effectiveness of the proposed method.*
Keywords: Digital redesign, Generalized bilinear transformation (GBT), Sampling period, Discretization parameter, Switched system, 2-delay (N-delay) control

1. **Introduction.** In real applications, many systems are continuous-time (analog), or in other words, described by differential equations due to their physical properties. To control such systems using computer generated signals, it is necessary to design a digital (discrete-time) controller for practical use (in fact, there is no computer which can generate absolutely continuous-time control signals). This is called *digital control* in general. It is well known that the methods of designing digital controllers can be classified into three categories [1] (Figure 1). The first method (route (a) in Figure 1) is the so-called *discrete design*, where a discrete-time counterpart is first obtained from the original analog system via some discretization method, and then a digital controller is designed for the discrete-time system. In this route, the inter-sample point behavior is always a problem that needs to be dealt with carefully. The second one (route (b) in Figure 1) is the so-called *sampled-data design* [2], where a digital controller is designed directly for the analog (continuous-time) system based on lifting technique. It is known that the design procedure in sampled-data design is generally very complicated and in some cases not feasible using the existing numerical computation tools [3].

What we focus on in this paper is the third method (route (c) in Figure 1), the so-called *digital redesign*, which is composed of two steps. The first step is called the *design step*, where a continuous-time controller is designed for the analog system. The second step is called the *implementation step*, where the continuous-time controller is discretized so as to obtain a discrete-time controller. There are many efficient and practical methods for the design step, while for the implementation step, the main methods are Tustin, ZOH equivalent, the Euler method and the matched pole-zero method [4]. Since these methods usually have to involve a process of trial and error repeating the design step and the implementation step, a one-way method guaranteeing the system performance has been desired. For this purpose, a new controller discretization method, the so-called *Generalized Bilinear Transformation (GBT)* was proposed [5, 6] to provide a class of discrete-time controller approximations parameterized by a parameter $\alpha \in (-\infty, \infty)$. The design step is the same as before, while the implementation (discretization) step involves an optimization with respect to the parameter α so as to guarantee the system performance including stability, step tracking, and/or H_∞ control, etc. [6]. Although there have been many other references in the literature studying optimization-based controller discretization (e.g., [7]), the parameter α in GBT adds an extra degree of freedom to the control system in consideration, which makes the closed-loop system very adaptive [6]. Other advantage of GBT has been pointed out in [5, 6] in more detail. For example, controllability and observability are invariant under GBT, and it may convert unstable poles (zeros) to stable poles (zeros).

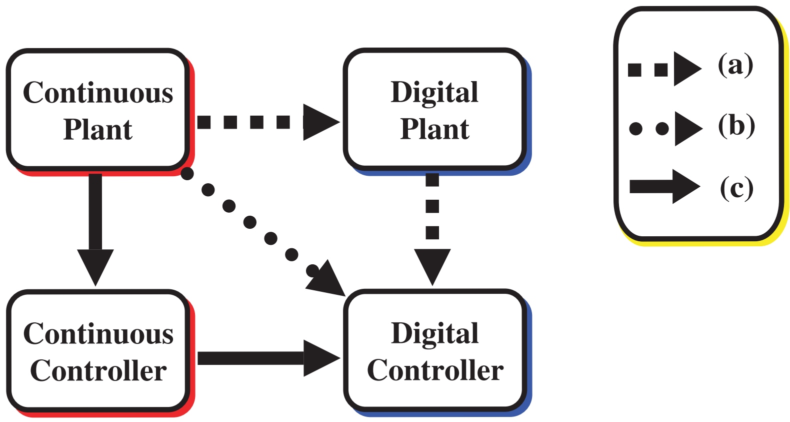

FIGURE 1. Three categories in digital controller design

In addition to the above mentioned theoretical advantage, it is also known that GBT is easy to implement and the discretization parameter is not difficult to adjust. However, we find through many simulation examples that it tends to result in large overshoot, which certainly is not a desirable behavior in real systems. To suppress the overshoot within the GBT, we need another routine of trial and error, and it usually results in a trade-off among various system performance.

Motivated by the above observation and simulation experience, we here aim to propose an approach focusing on the discretization parameter adjustment so as to overcome the overshoot problem. As can be seen later, there are two design methods in our approach. The first one is enlightened by the observation that some parameters lead to large overshoot but good convergence, while other parameters do not make the system stable but the overshoot (in a specified area) is small, and thus a combination of the two kinds of parameters may be desirable. The algorithm takes advantage of the existing results in switched systems, and proposes using piecewise discretization parameters in the same approximation of an integrator as in the existing GBT, together with a class of switching laws. The second method gets hint from the idea that a single parameter in the discretization may not be enough, and thus proposes to add an extra discretization parameter in the approximation. The detailed algorithm concerning how and where to input the extra parameter is based on the idea of using an inter-sample point operator, as has been used in the 2-delay (N-delay) control. It turns out that both design methods can improve the overshoot of the closed-loop system significantly.

The remainder of this paper is organized as follows. In Section 2, we give some preliminaries about GBT in digital control systems together with a simulation example. Section 3 establishes two design methods and algorithms concerning the discretization parameter, and two numerical examples are included to show effectiveness of the proposed design methods. Finally, Section 4 concludes the paper.

2. **GBT in Digital Control Systems.** A typical architecture of digital feedback control systems is described in Figure 2, where a digital controller is adapted to a continuous-time plant. Moreover, a sampler S is used to obtain discrete-time signals from the continuous-time states/outputs, and a zero-order hold (ZOH) H is used to convert the discrete-time input signals, generated by the digital controller K, into continuous-time inputs for the system. The solid line in Figure 2 denotes the domain of continuous-time signal (data) while the dashed line means discrete-time signal is dealt with.

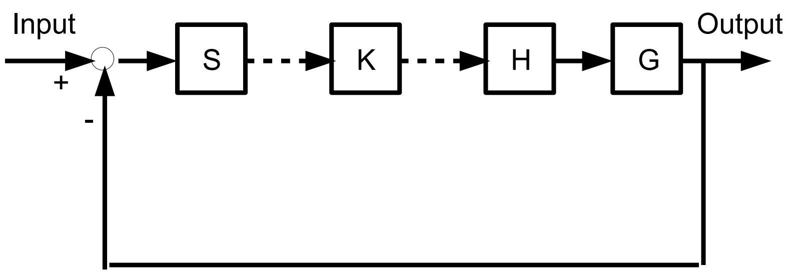

Figure 2. Digital feedback control systems

As also mentioned in the introduction, GBT is one method of implementing a continuous-time controller as a discrete-time one. The key idea of GBT is based on the trapezoidal approximation of an integrator. Given an integrator $\frac{1}{s}$ with input u and output y, the trapezoidal approximation of

$$y(kh+h) = y(kh) + \int_{kh}^{kh+h} u(\tau)d\tau \tag{1}$$

is

$$y(kh+h) = y(kh) + h\frac{u(kh+h)+u(kh)}{2}, \tag{2}$$

i.e., the integral is approximated by using the average value of $u(kh+h)$ and $u(kh)$. The idea of GBT [5] is to replace this average value with another positive-weighted combination of $u(kh+h)$ and $u(kh)$. More precisely, using a positive parameter $\alpha \in (0,1)$ to approximate the integral in (1) as

$$h\left(\alpha u(kh+h) + (1-\alpha)u(kh)\right) \tag{3}$$

and thus

$$y(kh+h) = y(kh) + h\left(\alpha u(kh+h) + (1-\alpha)u(kh)\right). \tag{4}$$

Then, the transfer function of (4) (in z transform) is

$$\frac{Y(z)}{U(z)} = h\frac{\alpha z + (1-\alpha)}{z-1}. \tag{5}$$

Since $\dfrac{Y(s)}{U(s)} = \dfrac{1}{s}$, it motivates us to introduce the approximation

$$\frac{1}{s} \approx h\frac{\alpha z + (1-\alpha)}{z-1} \tag{6}$$

or equivalently,

$$s \approx \frac{1}{h}\frac{z-1}{\alpha z + (1-\alpha)}\,. \tag{7}$$

Using the above approximation, for a pre-designed continuous-time controller denoted by the transform function $K_c(s)$, the GBT discretization method is to substitute (7) into $K_c(s)$ as

$$K_d(z) \triangleq K_c\left(\frac{1}{h}\frac{z-1}{\alpha z + (1-\alpha)}\right), \tag{8}$$

which is a digital controller with the discretization parameter α being adjusted later via some optimization method. As also described in [5], the realization from $K_c(s)$ to $K_d(z)$ is very simple. For example, if the state space representation of $K_c(s)$ is

$$\dot{x}(t) = A_K x(t) + B_K u(t), \quad y(t) = C_K x(t) + D_K u(t), \tag{9}$$

then the state space representation of $K_d(z)$ in (8) can be computed as

$$x[k+1] = A_{dK} x[k] + B_{dK} u[k], \quad y[k] = C_{dK} x[k] + D_{dK} u[k] \tag{10}$$

where

$$\begin{aligned} A_{dK} &= (I - \alpha h A_K)^{-1}\left[I + (1-\alpha)hA_K\right] \\ B_{dK} &= (I - \alpha h A_K)^{-1} h B_K \\ C_{dK} &= C_K (I - \alpha h A_K)^{-1} \\ D_{dK} &= D_K + \alpha C_K (I - \alpha h A_K)^{-1} h B_K. \end{aligned} \tag{11}$$

In [6], the range of the discretization parameter α is extended to $(-\infty, \infty)$, and thus both positive and negative weighted combinations of $u(kh+h)$ and $u(kh)$ are applicable.

As also remarked in [5, 6], when $\alpha = 0, \frac{1}{2}$ and 1, the discretized controller (8) is the forward Euler, Tustin and backward Euler approximation of $K_c(s)$, respectively. Thus, noticing the parameter's range is enlarged to $(-\infty, \infty)$, we have earned much more design degree of freedom with the parameter.

For example, consider the continuous-time plant and the controller described by

$$G(s) = \frac{10}{s^2 + s}, \quad K_c(s) = \frac{0.416s + 1}{0.139s + 1}, \tag{12}$$

which has been used in [6, 7] as an illustrative example. It is noted that the stability (the poles) of $G(s)$ here does not lead to stability and other performance in the feedback control system described in Figure 2.

When the sampling period is $h = 0.42$, the state trajectories of the closed-loop system, using the discretization method in [6, 7], are depicted in Figure 3, where the state trajectory of using the continuous-time controller is included for comparison and $\alpha = 0.25$ in the GBT controller.

It is clear from Figure 3 that compared with the digital controller in Keller and Anderson [7], the GBT controller results in fast convergence even with large sampling period, but the overshoot is quite large, which is not desirable in many real systems. This is the main issue to study in the present paper.

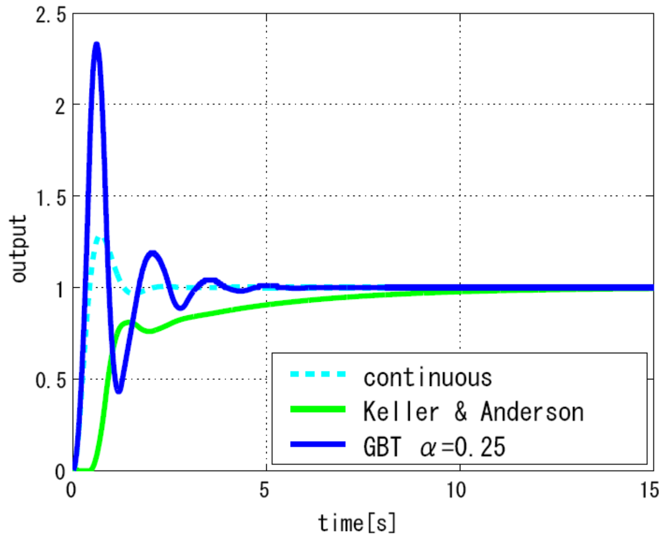

FIGURE 3. Step input responses comparison ($h = 0.42$)

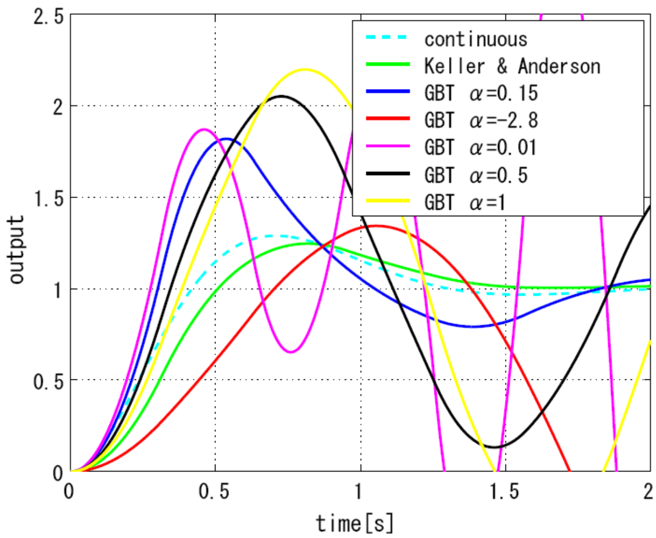

FIGURE 4. Step input responses with different α's

3. **Digital Redesign via Extension of GBT.** In this section, we propose to use multiple discretization parameters in GBT so as to deal with the overshoot problem mentioned in the above sections. The motivation is based on the observation that the closed-loop system with different discretization parameters has different performance (convergence, overshoot, etc.). For example, consider again the system (12). With the same sampling period, the state trajectories of the closed-loop system with several different α's are depicted in Figure 4. It is observed that when $\alpha = 0.01$ or $\alpha = 1$, the closed-loop system is unstable; when $\alpha = 0.15$, the closed-loop system is stable but the overshoot is quite large (about 0.8). Here, we first propose to use different parameter α on different time interval, and then propose to modify the approximation (1) with an extra discretization parameter, so as to obtain better performance improving the overshoot problem.

3.1. **Piecewise discretization parameter.** A simple way to consider different discretization parameter on different time interval is to use piecewise parameter. Obviously, the key issues in this case are: (1) what and how many parameters are used? (2) How to determine the active time period for each parameter? Certainly, these issues are related to each other in any system. It is impossible to provide a universal answer since there are infinite number of combinations, and here we aim to provide one practical method, which is expected to provide an answer to the above questions.

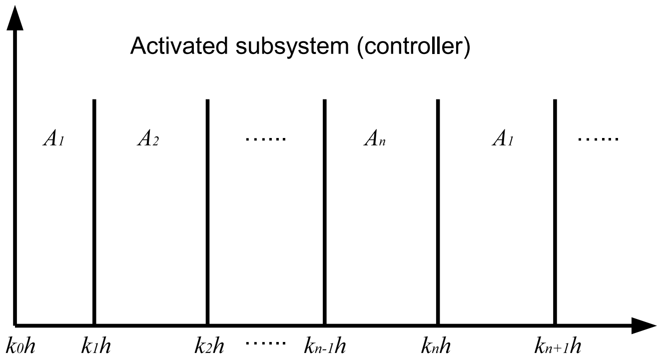

Figure 5. Switched system corresponding to piecewise discretization parameter

Given several discretization parameters, if we regard the closed-loop system with each parameter as a subsystem, then the entire system of using piecewise parameter is actually a switched system composed of the resultant subsystems. Thus, we can apply the rich theoretical results in switched systems to the present problem of switching among the discretization parameters.

Suppose that the sampling period is h, and n discretization parameters α_i $(i = 1, \cdots, n)$ are used in order on $[k_{i-1}h, k_i h)$ iteratively. Denote the resultant system matrix by $A_1, \cdots, A_n$. Figure 5 shows the time sequence in this case. Then, the entire system is described by

$$x[k+1] = \boldsymbol{A}_i x[k] \tag{13}$$

$$i = \begin{cases} 1 & (k_{0+m}h \leq t < k_{1+m}h) \\ 2 & (k_{1+m}h \leq t < k_{2+m}h) \\ \cdots & \cdots \\ n & (k_{n-1+m}h \leq t < k_{n+m}h) \quad (m = 0, n, 2n, \cdots) \end{cases}$$

where $k_1 < \cdots < k_n < \cdots$ are positive integers determining the switching time instants of parameters, and thus $k_{i+m}h - k_{i-1+m}h = \Delta_i h$ $(i = 1, \cdots, n)$ is the activation time period of the i-th subsystem (discretization parameter) during one iteration.

Motivated by the results in stability analysis of switched linear systems [8, 9], we obtain the following theorem.

Theorem 3.1. *Choose positive parameters λ_i's $(i = 1, \cdots, n)$ such that $\mid \lambda_i \boldsymbol{A}_i \mid < 1$. If the positive integers k_i's are adjusted satisfying*

$$\prod_{i=1}^{n} \left(\frac{1}{\lambda_i} \right)^{2\Delta_i} < 1, \tag{14}$$

then the entire system (13) is exponentially stable.

Proof: Since $\mid \lambda_i \boldsymbol{A}_i \mid < 1$ is equivalent to

$$(\lambda_i \boldsymbol{A}_i)^T (\lambda_i \boldsymbol{A}_i) < I, \tag{15}$$

$V(x) = x^T x$ is a common Lyapunov function for all systems whose system matrices are $\lambda_i A_i$ $(i = 1, \cdots, n)$. According to (15), the inequality

$$x^T[k]\boldsymbol{A}_i^T\boldsymbol{A}_i x[k] < \left(\frac{1}{\lambda_i}\right)^2 x^T[k]x[k] \tag{16}$$

holds for any nonzero $x[k]$. Noticing that $x[k+1] = \boldsymbol{A}_i x[k]$, we obtain that $x^T[k+1]x[k+1] < \left(\frac{1}{\lambda_i}\right)^2 x^T[k]x[k]$ and thus when the ith discretization parameter is active,

$$V(x[k_i]) < \left(\frac{1}{\lambda_i}\right)^{2\Delta_i} V(x[k_{i-1}]). \tag{17}$$

Therefore, evaluating the value of $V(x)$ from the start to the end of one iteration, we obtain

$$V(x[k_n]) < \prod_{i=1}^{n}\left(\frac{1}{\lambda_i}\right)^{2\Delta_i} V(x[k_0]). \tag{18}$$

The condition (14) guarantees that $V(x[k_n]) < \mu V(x[k_0])$ $(\mu < 1)$, which implies the value of the Lyapunov function $V(x)$ decreases at a specified rate strictly on each iteration, and thus the system state converges exponentially. This completes the proof. □

Remark 3.1. *The number n and the value of the discretization parameters α_i's are determined by observing the behavior of the resultant closed-loop system with certain parameter. Since the condition (14) does not depend on the order of (λ_1, Δ_1), (λ_2, Δ_2), $\cdots$, (λ_n, Δ_n), the activation order of each subsystem (parameter) in Figure 5 can be arbitrary, provided that the dwell (activation) time $\Delta_i h$ does not change.*

Remark 3.2. *To solve (or check) the condition (14) with respect to Δ_i, it is more practical to use the equivalent inequality*

$$\sum_{i=1}^{n}\Delta_i \ln \lambda_i > 0. \tag{19}$$

Concerning the choice of each λ_i, we can choose $\lambda_i > 1$ for (Schur) stable A_i's, while $\lambda_i < 1$ for (Schur) unstable A_i's. Actually, since Δ_i's are positive integers, it is easy to tell from (14) or (19) that there is at least one $\lambda_i > 1$. This implies that the condition in the theorem requires at least one stabilizing controller. Moreover, it is observed from (18) that the convergence of the system is dominated by $\prod_{i=1}^{n}\left(\frac{1}{\lambda_i}\right)^{2\Delta_i}$. Thus, when the main focus is the convergence issue, one should choose the parameters such that $\prod_{i=1}^{n}\left(\frac{1}{\lambda_i}\right)^{2\Delta_i}$ is as small as possible.

The above two remarks suggested precise and practical ways of choosing the parameters λ_i's. It can be seen that there is still much freedom satisfying the condition in the theorem, and thus more control specification can be considered. In that case, certain kind of optimization algorithm may be necessary for choosing the parameters.

Remark 3.3. *When both stable and unstable A_i's exist,* [8, 9] *proposed a more flexible switching method, which specifies an average dwell time of all subsystems and limits the activation time ratio between unstable subsystems and stable ones. See* [8, 9] *and the extended approach in* [10] *for more detailed discussion.*

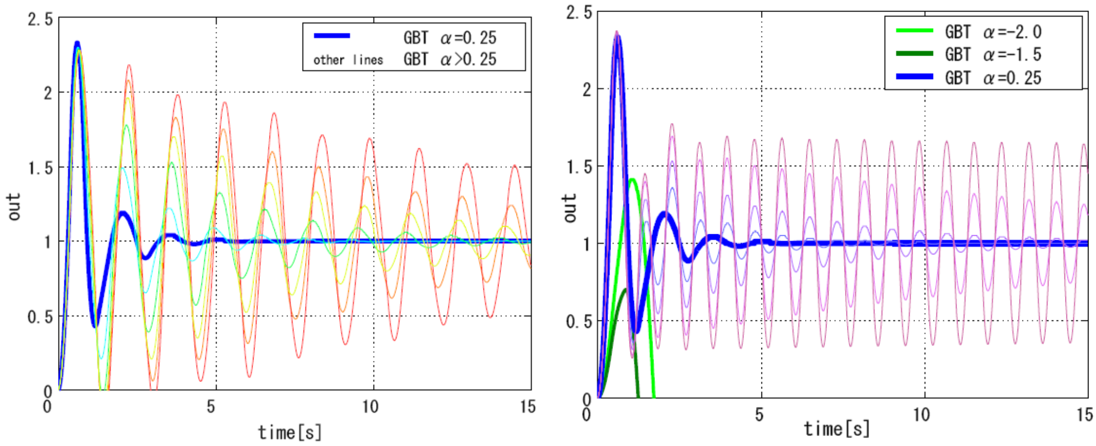

FIGURE 6. Step responses when $\alpha \geq 0.25$ (left) and when $\alpha \leq 0.25$ (right)

In the end of this subsection, we provide an example. Again, consider the system and the continuous-time controller given by (12), and set the sampling time period to $h = 0.42$ as before. Then, using various discretization parameters α, the performance (step response) of the resultant system changes correspondingly. Figure 6 depicts the state trajectories, from which we observe that the settling time is the shortest when α is around 0.25 but the overshoot is almost twice the objective output.

Now, our objective is to decrease the overshoot while keeping almost the same convergence. After observing Figure 6 with various discretization parameters, we find that when $\alpha = -1.8$, the closed-loop system is unstable but the rise time is almost the same and the overshoot is smaller. In addition to the observation in the case of $\alpha = 0.25$, it motivates us to use $\alpha = -1.8$ first and switch to $\alpha = 0.25$ when the negative overshoot becomes large, and repeat the switching.

According to Theorem 3.1 and its condition, we set $\alpha_1 = -1.8$, $\alpha_2 = 0.25$ to obtain two subsystems. It is easy to confirm that A_1 is unstable and A_2 is stable. We then choose $\lambda_1 = 0.5$ and $\lambda_2 = 1.4$ such that $| \lambda_1 \boldsymbol{A}_1 | < 1$, $| \lambda_2 \boldsymbol{A}_2 | < 1$. Setting $k_0 = 0$ in the condition (14) to obtain

$$\left(\frac{1}{0.5}\right)^{2\Delta_1} \left(\frac{1}{1.4}\right)^{2\Delta_2} = 0.51^{k_2} 7.84^{k_1} < 1, \tag{20}$$

we can easily find one solution $k_1 = 2$, $k_2 = 30$. Using these values in the switching sequence of Figure 5, we obtain the step input response of the entire closed-loop system in Figure 7 (the bold solid line). Compared with the case of using single discretization parameter, the overshoot is suppressed while the convergence property does not degenerate much.

3.2. **Extra discretization parameter.** In this section, we propose another approach to improving the overshoot problem by focusing on the approximation (1). More precisely, we consider a new approximation with an extra discretization parameter so that various additional control performance including the overshoot one can be dealt with.

In addition to the integral approximation in (1), we add another term concerning the inter-sample control input $u(kh + h/2)$ together with a weighting parameter β to obtain

$$y(kh + h) = y(kh) + h \left[(\alpha - \beta) u(kh + h) + \beta u \left(kh + \frac{h}{2} \right) + (1 - \alpha) u(kh) \right]. \tag{21}$$

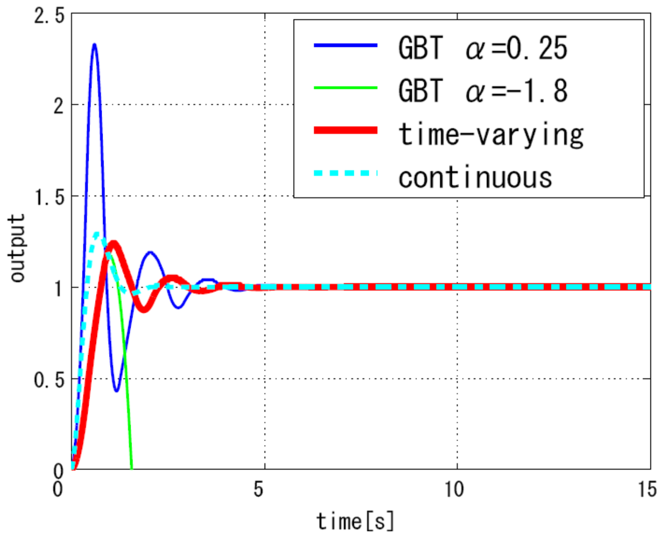

FIGURE 7. Performance improved with piecewise discretization parameter ($\alpha_1 = -1.8$, $\alpha_2 = 0.25$)

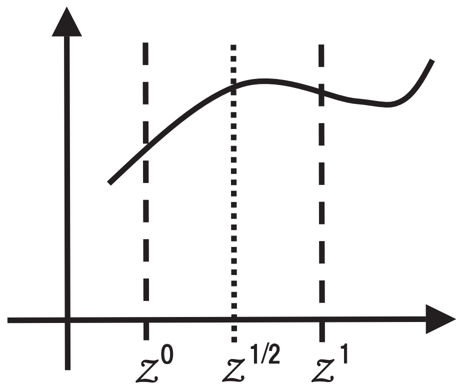

FIGURE 8. Inter-sample point operator $z^{\frac{1}{2}}$

Similar to the GBT derived in Section 2, we obtain the discretization approximation

$$s \approx \frac{1}{h} \frac{z-1}{(\alpha-\beta)z + \beta z^{\frac{1}{2}} + (1-\alpha)} \tag{22}$$

where $z^{\frac{1}{2}}$ is an operator generating the data at inter-sample points, as described in Figure 8.

However, the sampling periods of the sampler and the digital controller are generally same, and thus it is physically not easy to obtain the value of $z^{\frac{1}{2}}$. This difficulty can be overcome by using the so-called 2-delay control input. As shown in Figure 9, we set up two samplers, which have the same sampling period h but take sample inputs with a time delay. Here, since the inter-sample point inputs are desired, we set the time delay as $\frac{1}{2}h$. Although there is some restriction of the systems when applying 2-delay control input, we find this approach is effective for a broad class of digital systems. Actually, the 2-delay control method can also be extended to the case of N-delay control [12] when more complex performance is desired.

Basically, the two methods in Sections 3.1 and 3.2 are different, and it is not easy to tell which one is better. The extra parameter method requires the inter-sample point

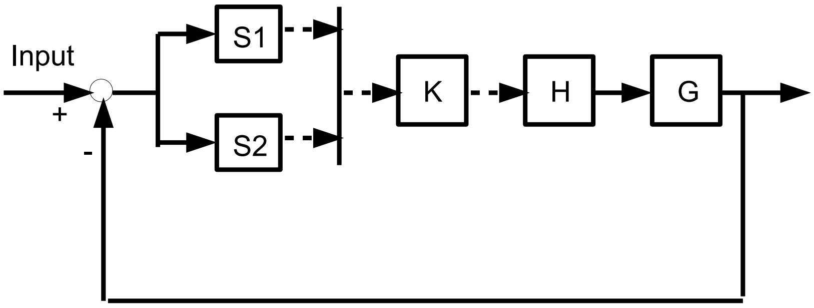

FIGURE 9. Digital system with 2-delay control input

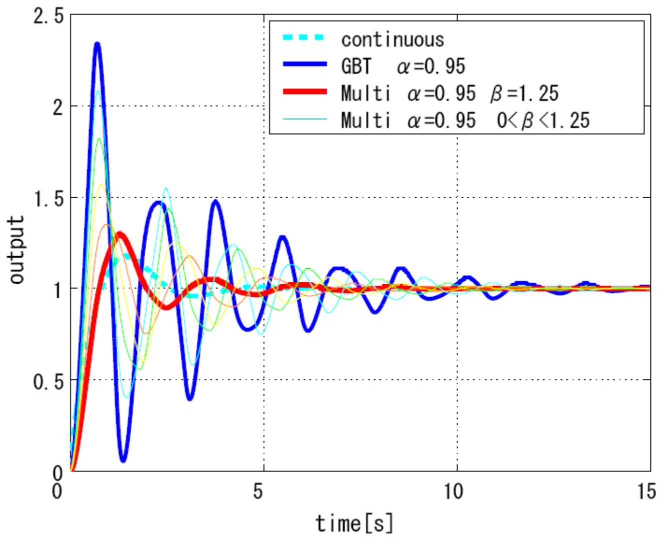

FIGURE 10. Performance improved with extra discretization parameter

data, which can be regarded as a limitation in real systems. Concerning the performance, according to our computation experience, both methods lead to quite satisfactory results, but with different number of trial and errors.

Next, we provide an example showing effectiveness of the above method. Consider the continuous-time system and the PID controller

$$
\begin{aligned}
G(s) &= \frac{1}{s^2+5s+4} \\
K_c(s) &= 10.2\left(1+2.38\frac{1}{s}+0.28s\right).
\end{aligned}
\tag{23}
$$

Suppose that the sampling period is $h = 0.60$. We observe that when $\alpha = 0.95$, the closed-loop system is stable with a short rise time, but the overshoot is quite large. Then, we introduce an extra parameter $\beta = 1.25$ by some trial and error, and use the approximation (22) to obtain the digital controller. As shown in Figure 10 (the bold solid line), the performance of the closed-loop system is satisfactory.

4. **Conclusion.** In this paper, we have considered the digital redesign problem for digital feedback control systems with comparatively large sampling period. Motivated by the observation that the existing GBT approach tends to result in large overshoots, we have proposed two design methods concerning the adjustment of discretization parameters to

improve the overshoot problem. It is more flexible to combine these two methods in real applications, when the inter-sample point data is available.

There are several issues in our future research. Although we focused on improving the overshoot problem with similar (large) sampling period, it is an interesting and challenging problem to obtain large sampling period via switching method. Also, when additional control specification is desired, it is practical to consider how many parameters should be used, and how to switch among them. In the case of considering disturbance attenuation, the switching strategy proposed in [13] for $\mathcal{L}_2$ gain analysis of switched linear systems may be effective. Moreover, although the overshoot problem can be observed directly in the trajectory graph, other performance usually should be evaluated in a quantitative manner. This may lead to difficulty in determining (choosing) the parameter and the switching instants. Another practical extension is to deal with decentralized stochastic large-scale systems with time-varying time delays (for example, [14]) by using the present framework. Combing quantization for networked control systems [15] with digital control design and implementation is also an open and challenging issue.

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